Epistemic relevance and epistemic actions

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Abstract An operational and informational semantics for the ternary relation *R* is explored as a framework for modeling informational relevance. We extend this framework into robustly epistemic terrain. We take a new perspective on the problem of logical omniscience, using informationalised operational semantics to model the properties of the epistemic actions that underpin the epistemic relevance of certain explicit epistemic states of an epistemic agent as that agent executes said actions.

Keywords: epistemic actions, logical omniscience, informativeness, relevance logics, structural rules

Introduction

The problem of logical omniscience is the problem faced by epistemic modeling given that basic epistemic logics assume that the epistemic agents are logically omniscient, but we are not. This is a hard problem. The scandal of deduction is the failure of philosophy to give a sensible account of how it could be that deductive reasoning can be informative for us given that such inferences deliver zero information. The scandal of deduction has a straightforward answer, and this answer illustrates a way in which the problem of logical omniscience might be overcome. The answer to the scandal is best illustrated via a walk-through on general (non-logical) omniscience, and the information that we get from our empirical environment.

We get information from our environment either distally via direct observation, or indirectly via announcements. Examples are familiar from the philosophical canon. Consider *grass is green*, *snow is white*, or *there are one hundred and one dormice in the room next door*. In order to get information from our environment, we need to

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perform certain *epistemic actions*. These are actions such as the aforementioned observations or announcements.[1] We need to perform epistemic actions of these types on account of our not being omniscient. If we were omniscient, then we would not need to perform such actions because we would have automatic, effortless access to all information in our environment simply by definition. This is just what it means to be omniscient. We may, with a little poetic license, think of omniscience as the *limit* of the epistemic action of observation. Omniscience is the epistemic state achieved when further observation could not add any more information to our information base.

Just as we are not omniscient, neither are we *logically omniscient*. In order to use the information that we get from our environment, we need to reason with it. Such reasoning is an epistemic action of a cognitive sort, insofar as it is an action of the mind. We need to carry out such epistemic actions in order to bring the information corresponding to both logical theorems, as well as the logical consequences of our environmentally acquired information, into our information base. Analogous with the point made about omniscience and observations above, if we were logically omniscient, then we would not need to carry out reasoning-style epistemic actions because we would have automatic, effortless access to all logical theorems as well as all logical consequences of the information gotten from our environment. This is just what it means to be logically omniscient. Again, with a little poetic licence, we may think of logical omniscience as the limit of the epistemic action of deductive reasoning. Logical omniscience is the epistemic state achieved when further deduction could not add any more information to our information base.

To be sure, when we speak of an epistemic agent being omniscient in the general sense, we often take this to imply that the agent is logically omniscient also. Nonetheless, these two types of omniscience remain conceptually distinct. There is, to be sure again, a commonly recognized priority of sorts between the two omniscience types. It is not particularly useful to think about an omniscient, but non-logically-omniscient agent. Such an agent might not be able to do all that much with the information that it got from its environment, if that agent lacked suitable logical acumen.[2] Being omniscient entails being in possession of a great deal of information, hence some heavy duty logical acumen would be required to handle it.

When it comes to modeling epistemic actions of the observation sort, the agents being modeled are assumed often to be logically omniscient for just this reason (see van Ditmarsh et al. (2008)). By abstracting away from the cognitive epistemic actions which underpin logical information handling, such frameworks—standard dynamic epistemic logic (DEL) for example—may concentrate on the properties of

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[1] Announcements and observations may be run together as a single type of epistemic action if you assume that announcements are always truthful, always believed, and always non-noisy, van Ditmarsh et al. (2008).

[2] This point is similar to the one made by Frege in his letter to Jourdain. Frege entertains an agent who is able, in principle, to comprehend every atomic sentence, but does not have the ability to execute any semantic composition. Given language's essential productivity, such an ability is, according to Frege, of little general interest. I am indebted to an anonymous referee for bringing this to my attention.

the information-updates resulting from observation-type epistemic actions. In such frameworks, the epistemic agent is assumed to be logically omniscient, or ideally rational, and the cognitive epistemic actions executed by the agent get "black boxed."[3]

This essay is an attempt to say something philosophically substantial about the nature of the epistemic actions which underpin logical information handling—to shine some light inside the black box.[4]

1 Epistemic relevance and relevance logics

Both information itself as well as epistemic actions may be *epistemically relevant*. Some information is epistemically relevant for an agent if it is relevant to the agent's epistemology, where by this we mean the agent's knowledge or beliefs. For example, if you need to know how many bottles of wine you might need for your dinner party, then the number of guests is epistemically relevant. Similarly, if you have the information that the terrorist cell will attack either the Sydney Harbour Bridge or the Sydney Opera House, then the information that the terrorist cell will not attack the Sydney Harbour Bridge is epistemically relevant to your counter-terrorist plans.

An epistemic action will be epistemically relevant for an agent if the execution of the action gets information for the agent such that this information is epistemically relevant in the manner described above. For example, the announcement from each of your dinner party's invitees that they are able to attend the party will be a collection of epistemically relevant epistemic actions. Similarly, an observation of the terrorist cell's moving their personnel away from the Sydney Harbour Bridge is epistemically relevant. Both the dinner party and terrorist cell examples assume that you are able to reason with, or integrate, or logically handle the information that you got from the announcement and observation actions. As we noted in the previous section, this handling of information in a logical manner is an epistemic action of an internal, cognitive sort. Logically handling or reasoning with information will be epistemically relevant for an agent if the execution of such reasoning gets information for the agent such that this information is epistemically relevant.

Both the nature of epistemic relevance and the nature of the cognitive epistemic actions which underpin deductive reasoning could do with clarification and elaboration. We can find both of these with some help from *relevance logics* (see Mares (2004) for a canonical introduction).

That relevance logics provide a logical framework for epistemic relevance and epistemic actions is at the very least not obvious. Such logics are neither thought of as particularly epistemic, nor as dynamic (and actions, epistemic or otherwise, are dynamic if anything is). To see how it is that we might be justified in thinking

[3] Assuming logical omniscience for the epistemic agents in one's model makes perfect sense insofar as one wants to idealize away from variables.

[4] The motivation here is similar to that of (Duc, 1997). The difference is that Duc has models for what the agent knows *after* she has executed some rule of inference or other, whereas here we will be modelling the properties of the epistemic actions which underpin the execution of such rules.

of relevance logics as being both, we will skip the detailed nomenclature of the logics' syntactic/proof theoretic properties and motivations, and go directly to the semantics.

In relevance logics, a *relevance frame* **F** is a pair $\langle S, R\rangle$ consisting of a set $x, y, z, \ldots \in S$ of points of evaluation, and a ternary relation R on this set. A *relevance model* **M** is a pair $\langle \mathbf{F}, \Vdash\rangle$ consisting of a relevance frame **F** and an evaluation relation $\Vdash$ which holds between the points of evaluation in S and formulas $\phi, \psi, \ldots$.

We may now state the evaluation conditions given by relevance logic for the conditional $\phi \rightarrow \psi$ as follows.

$$x \Vdash \phi \rightarrow \psi \quad \textit{iff} \quad \forall y, z : Rxyz, \text{ if } y \Vdash \phi, \text{ then } z \Vdash \psi. \tag{1}$$

(1) is still slightly opaque. What are the points of evaluation $x, y, z, \ldots$, and what does R mean?

The points of evaluation work just like possible worlds, except that in the present case they may be both inconsistent and incomplete. It is common practice to speak of the points of evaluation as *information states*, since there is no obvious constraint on a body of information that it be complete or consistent. Making sense of such information states insofar as we want them to correspond to something in the real world is the task of section 2 below.

How to make sense of $Rxyz$ is an infamous issue. We might understand $Rxyz$ as something like "*if you combine the things which are true at x with the things which are true at y then you get the things which are true at z*." This is a good start, but does $x \Vdash \phi$ mean that ϕ is true at x? It does not, not quite. Given that inconsistent propositions may hold at points, that is, given that we may have $x \Vdash A \wedge \neg A$, understanding $\Vdash$ as "true at" is a little too crude.

Instead, we may understand $x \Vdash \phi$ as "*x carries/stores the information that* ϕ." In this case, $Rxyz$ comes out as "*if you combine the information carried by/stored at x with the information which is carried by/stored at y then you get the information which are carried by/stored at z*." This is an improvement over a "true of" understanding, and it puts us in a position to use relevance frames (and their corresponding models) in order to understand both epistemic relevance and cognitive epistemic actions.[5]

2 Epistemic relevance and epistemic actions

Following Dunn (2015), we will take the partial order of information inclusion, $\sqsubseteq$, to indicate *information relevance*.[6] In this case, $x \sqsubseteq y$, "*the information at x is*

[5] See Mares (1996) and Restall (1996) for the fine-grained details involved in "informationalising" the ternary relation R. See also Dunn and Hardegree (2001).

[6] The role of a partial or pre-order in the Routley–Meyer semantics for relevance logic is well-known and explored in some detail in (Bimbó and Dunn, 2008, Ch. 2).

included in the information at y," means that the information at x is *relevant to* information at y.

This does not seem to be too much of a stretch. If the information at x is included in the information at y, then the information at the former seems relevant to the information at the latter on account of the inclusion itself. The information at y takes the information at x to be relevant because the information at y just is an informational extension of the information at x.

The informational relevance indicated by $x \sqsubseteq y$ is *non-contextual* relevance insofar as the relevance of x to y does not depend on any further information (or further informational context, as we might say). Suppose that $x \Vdash A$ and $y \Vdash A, B$. In this case we might have it that $x \sqsubseteq y$.[7] However, suppose instead that $y \Vdash A$ and $z \Vdash B$. Is it the case that we might have it that $y \sqsubseteq z$? Not as things stand, which is to say not without some further informational context.

Such further informational context may be given as follows. Suppose that we have $x \Vdash A \to B$. In the context of x (and given that $y \Vdash A$ and $z \Vdash B$ as specified in the paragraph above), it is the case that $y \sqsubseteq z$. This is just to say that if we take the information in state x together with the information in state y, then these two information states, when taken together, carry information which is relevant to the information in state z. We may represent this taking together of, or combination of, two information states with a binary composition operation on information states, $\bullet$. Given that $x \Vdash A \to B$, $y \Vdash A$ and $z \Vdash B$, we have it that $x \bullet y \sqsubseteq z$. In other words, given the information carried by states x and y, their combination is relevant to the information carried by state z. Moreover, the very act of combining x and y is itself informationally relevant to z. This is because it is the operation of combining x and y which bring the information at both states together. Sans such an operation, the information at x and y are separate informational entities, neither of which, either considered independently or non-contextually, are informationally relevant to z.

We may now give a more thoroughgoing explanation of $Rxyz$. We may understand $Rxyz$ as $x \bullet y \sqsubseteq z$. In this case, our relevance frame becomes an *information frame* **I**, which is a triple $\langle S, \sqsubseteq, \bullet \rangle$. Our relevance model becomes an information model $\mathbf{M}_I$. Given this much, (1) comes out as:

$$x \Vdash \phi \to \psi \quad \textit{iff} \quad \forall y, z : x \bullet y \sqsubseteq z, \text{ if } y \Vdash \phi, \text{ then } z \Vdash \psi. \tag{2}$$

With the full informational relevance interpretation of the relevance semantic conditions for the conditional in hand, we are in a position to see how it is that relevance frames have a role to play with regard to understanding our target phenomena—epistemic relevance and cognitive epistemic actions.

We stated above that making sense of the information states insofar as we want them to correspond to something in the real world was the present task. By making

[7] This is not guaranteed, since there is no sensible requirement on an epistemic state that the state in question be itself epistemically relevant to another epistemic state that subsumes the information carried by the original state. For example, my knowing that *grass is green* at some point in time does not have to be an epistemically relevant episode to every future epistemic state or action involving the information that grass is green. That is, our epistemic states are not totally ordered.

such sense, we will be on our way to addressing the issue posed at the end of the previous paragraph. Here is the suggestion:

We may understand the information states $x, y, z, \ldots$ to be states of explicit knowledge/belief of an epistemic agent, in other words, as *explicit epistemic states*.

By understanding the information states to correspond to explicit epistemic states of an agent, we have a direct link between information relevance, on the one hand, and our target phenomena of epistemic relevance, on the other. Suppose again that $x \Vdash A$ and $y \Vdash A \wedge B$. Now the former states that some agent α knows/believes explicitly that A, with the latter now stating that α knows/believes explicitly that $A \wedge B$.[8] In this case, $x \sqsubseteq y$ states that the agent's explicit epistemic state x is non-contextually epistemically relevant to their explicit epistemic state y.[9] But it is with *contextual* epistemic relevance that things get interesting.

Suppose that α is in the states $x \Vdash A \rightarrow B$, and $y \Vdash A$. This alone is insufficient for α to be in the state z such that $z \Vdash B$. For α to be in a state z such that $z \Vdash B$, α needs to *combine* the information in her states x and y. This is just to say that having explicit knowledge/belief of/in premises is insufficient for explicit knowledge/belief of/in conclusions. In order for α to get to z, she has to *think about things in the right way*. To think about things in the right way just is to combine the information encoded by the premises in such a manner that the result of this combination will make the information encoded by the conclusion explicit to α. The act of combining explicit epistemic states is just that, an act, or action. And it is such epistemic actions that underpin logical information handling, or the *process* of deductive reasoning itself.

At the end of the introduction we said that we were working towards saying something philosophically substantial about the nature of these epistemic actions. Information frames allow us to now do so.

$x \bullet y$ is a representation of the very epistemic action that we are looking for. Given that $x \Vdash A \rightarrow B$ and $y \Vdash A$, then given that $z \Vdash B$, it will be the case that $x \bullet y \sqsubseteq z$. Given our understanding of information states as explicit epistemic states, and of $\sqsubseteq$ as epistemic relevance, and of $\bullet$ as the epistemic action of combining such states, $x \bullet y \sqsubseteq z$ says something significant. It says that α's being in the explicit epistemic states x and y, *and* the execution by α of the epistemic action of combining these states, are *both* epistemically relevant for α's being in the epistemic state z.[10]

This is exactly what we are after. Given that we have a sensible framework for representing the cognitive epistemic actions which underpin deductive reasoning, the task now is to use this framework to say something philosophically substantial

[8] Of course, we could write "α knows/believes explicitly that A" as $x \Vdash_{\alpha} A$ or some such, but typographical rigour has a tendency to get in the way of readability.

[9] As well might be the case, given that both x and y carry A.

[10] Although cognitive epistemic actions may, and often do, involve the combination more that two premises, the treatment of the two-premise case is privileged on several fronts. Firstly, it is the simplest possible case. Given this, any model of cognitive epistemic actions needs to be shown to handle such cases before being applied to more complex cases. Secondly, it seems to be at least plausible that the majority of deductive episodes do proceed via two-premise combinations. Witness the standard natural deduction rules and classical syllogisms as examples. A third reason is simply that the two-premise case is hard enough.

about such actions. In particular, what properties might such epistemic actions possess which preserve epistemic relevance? In other words, what properties might an agent's cognitive epistemic actions possess such that the properties guarantee that the agent will arrive in the correct epistemic state?

3 Preserving epistemic relevance

The properties that an agent's cognitive epistemic actions will need to possess such that these properties guarantee that the agent will arrive in the correct epistemic state will vary. Their variance will depend upon the logical form of the information that is being handled by the epistemic action itself. We can capture the nature of these form-contingent action properties with *structural rules*.

A structural rule tells us what structural changes may be made to the body of information being processed, whilst preserving the given output of that same act of processing. Let's start with four basic structural rules, *Association*, *Commutation*, *Contraction* and *Weakening*. Where $\Longrightarrow$ is if-then in the metalanguage,

$$w \bullet (x \bullet y) \sqsubseteq z \Longleftrightarrow (w \bullet x) \bullet y \sqsubseteq z \qquad \text{(Association)}$$
$$x \bullet y \sqsubseteq z \Longrightarrow y \bullet x \sqsubseteq z \qquad \text{(Commutation)}$$
$$x \bullet x \sqsubseteq x \qquad \text{(Contraction)}$$
$$x \bullet y \sqsubseteq z \Longrightarrow x \sqsubseteq z \qquad \text{(Weakening)}$$

Association tells us that, given a sequence of information states, the order of pairwise composition within that sequence makes no difference to informational output.[11] Commutation tells us that given a pairwise composition of information states, the order of the information states in the pair being composed makes no difference to informational output. Contraction tells us that the composition of two information states that carry the same informational payload outputs no more information than that carried by one of the states. Weakening tells us that we can get the same informational output if we weaken the epistemically relevant information states.

Given that we are understanding the information states as explicit epistemic states, the composition operation as the epistemic action of combining such states, and the partial order of informational inclusion as epistemic relevance, then the epistemic action contexts in which the structural rules hold or fail become salient. They become salient because they specify the properties that said epistemic actions need to possess with regard to guaranteeing epistemic success.

We may begin by considering cases at the level of abstraction where the agent's epistemic states carry information of either atomic or conditional form. In this case,

[11] Note that Association is given here in its readable, abbreviated form. The full form of Association is $\exists u((x \bullet y \sqsubseteq u) \wedge (w \bullet u \sqsubseteq z)) \Longleftrightarrow \exists t((w \bullet x \sqsubseteq t) \wedge (t \bullet y \sqsubseteq z))$. This makes sense if you think about it. In the abbreviated form above, we are merely cutting out explicit reference to the states u and t, which are the results of composing w and x on the one hand, and x and y on the other, respectively.

α's explicit epistemic state x may be such that either $x \Vdash p$, or $x \Vdash p \to q$. In other words, α knows/believes explicitly some information which may be of either two forms. In this case, we have three possible scenarios given α's epistemic action $x \bullet y$, or, with a bit of a push, three types of epistemic actions. Both epistemic states may carry atomic information, or one epistemic state may carry information of atomic form and the other of conditional form, or both epistemic states may carry information of conditional form. Following Dunn (2015), we will call the first scenario the *Data Combining* (DC) interpretation, the second scenario the *Program Applied to Data* (PD) interpretation, and the third scenario the *Program Combining* (PC) interpretation.[12] So we have things as follows (reading " : " as *such that*, and "$z \Vdash p, q$" as shorthand for "$z \Vdash p$ and $z \Vdash q$").

$$\text{If } x \Vdash p \text{ and } y \Vdash q, \text{ then } x \bullet y \sqsubseteq z : z \Vdash p, q \tag{DC}$$

$$\text{If } x \Vdash p \to q \text{ and } y \Vdash p, \text{ then } x \bullet y \sqsubseteq z : z \Vdash q \tag{PD}$$

$$\text{If } x \Vdash p \to q \text{ and } y \Vdash q \to r, \text{ then } x \bullet y \sqsubseteq z : z \Vdash p \to r \tag{PC}$$

The consequences for the structural rules given these three epistemic action scenarios are interesting insofar as we are using the structural rules to specify the epistemically salient properties of the epistemic actions themselves.

Let's start with Association.

$$w \bullet (x \bullet y) \sqsubseteq z \Longleftrightarrow (w \bullet x) \bullet y \sqsubseteq z \tag{Association}$$

Association fails for some epistemic actions in PD scenarios. Consider the following explicit epistemic states w, x, y, z of α.

$$\begin{aligned} w &\Vdash q \to r \\ x &\Vdash p \\ y &\Vdash p \to q \\ z &\Vdash q \end{aligned} \tag{3}$$

In its left to right hand direction, Association fails for (3). This is just to say that although the epistemic action $w \bullet (x \bullet y)$ is epistemically relevant for α with respect to α's being in state z in a PD type epistemic action, that is to say, although we have it that $w \bullet (x \bullet y) \sqsubseteq z$, we do not have it that $(w \bullet x) \bullet y \sqsubseteq z$. Composing the information carried by w and x ($q \to r$ and p respectively) with PD type epistemic actions is an illegitimate epistemic action insofar as it will not get the agent anywhere, epistemically speaking. The result will not be anything which may be composed with the information carried by the α's state y ($p \to q$) such that is may be used to get α into

[12] Dunn uses "data" to refer to static information p, q, etc., and "programs" to refer to dynamic information, or conditionals, $p \to q$, etc. As we will see in section 4 below, agents may treat programs *as* data.

state z. Via similar reasoning, we can see that Association will fail in its right to left hand direction where we have it that $w \Vdash p \to q$, $x \Vdash p$, $y \Vdash q \to r$ and $z \Vdash r$.[13]

However, there is no failure for Association for epistemic actions involving PC scenarios. Consider any three epistemic states w, x, and y, such that each state carries information of composable conditional form. In this case, any output state z such that the epistemic action is epistemically relevant for α with respect to z (i.e., $(w \bullet x) \bullet y \sqsubseteq z$) will be preserved under Association. Consider the following explicit epistemic states of α.

$$\begin{aligned} w &\Vdash p \to q \\ x &\Vdash q \to r \\ y &\Vdash r \to s \\ z &\Vdash p \to s \end{aligned} \tag{4}$$

Association holds for (4), as it will for any PC scenario where the information states carry information with composable conditional form.[14]

In contrast with Association, however, Commutation holds for epistemic actions consisting of PD scenarios, but fails for those consisting of PC scenarios.

$$x \bullet y \sqsubseteq z \Longrightarrow y \bullet x \sqsubseteq z \tag{Commutation}$$

Consider again the epistemic states specified by (3). A thoroughgoing application of Commutation to (3) would give us the following.

$$w \bullet (x \bullet y) \sqsubseteq z \Longrightarrow (y \bullet x) \bullet w \sqsubseteq z \tag{5}$$

Given the epistemic states of α specified by (3), (5) holds (as is checked easily). In fact, Commutation will hold for any PD type collection of epistemic states whatsoever. This is because for any arbitrary pairwise composition of two pieces of information such that one piece is the input of the other piece, the composition will be order invariant. This is just a slick way of saying that for any two pieces of information such that one is of form A and the other is of form $A \to B$, the order of their composition is irrelevant insofar as deriving B is concerned, and similarly of course for the order of the epistemic states being composed by the relevant epistemic action.

However, Commutation fails for epistemic actions consisting of PC scenarios. Consider a simplified version of the scenario specified by (6).

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[13] There is a lot to say here about dynamic negation and negative information. One way to go is to say that there is a null object $\mathbf{0}$ such that $x \Vdash \mathbf{0}$ for no x. The way is then clear to define a dynamic negation $A^{\mathbf{0}}$ in terms of $A \to \mathbf{0}$, which will type information of the type that can never be combined with information of type A. Classical and other static negations rule out truth, whilst dynamic negations rule out certain operations or combinatorial procedures. See Dunn (1993), Dunn (1996), and Sequoiah-Grayson (2009).

[14] Since, as the category theory folks are fond of saying, "Arrows associate!"

$$\begin{aligned} x &\Vdash p \rightarrow q \\ y &\Vdash q \rightarrow r \\ z &\Vdash p \rightarrow r \end{aligned} \tag{6}$$

We have it that $x \bullet y \sqsubseteq z$. If Commutation held here, then we should have it that $y \bullet x \sqsubseteq z$, but this is not the case. This latter epistemic action is not epistemically relevant for α's being in the epistemic state z at all (since $(q \rightarrow r) \circ (p \rightarrow q)$ is the wrong order insofar as combining dynamic information is concerned).

The following related example emphasizes this point.

$$\begin{aligned} x &\Vdash p \rightarrow q \\ y &\Vdash q \rightarrow p \\ z &\Vdash p \rightarrow p \end{aligned} \tag{7}$$

With (7), we have it that $x \bullet y \sqsubseteq z$ also. But we do not have it that $y \bullet x \sqsubseteq z$. $y \bullet x$ results in a state $w \Vdash q \rightarrow q$, and $p \rightarrow p \neq q \rightarrow q$![15]

Consider Contraction.

$$x \bullet x \sqsubseteq x \tag{Contraction}$$

Contraction fails for epistemic actions of PC types in general, although it does hold for some special restricted cases. These cases are those where the antecedent and consequent of the relevant conditional encode the same information, as carried by the following explicit epistemic state.

$$x \Vdash p \rightarrow p \tag{8}$$

Contraction is preserved by epistemic actions that combine information of the type specified by (8), since the epistemic action in question is epistemically relevant to α's knowing explicitly that $p \rightarrow p$. Of course the epistemic action might well be *redundant*, but that is neither here nor there.

However, consider the following explicit epistemic state.

$$x \Vdash p \rightarrow q \tag{9}$$

Contraction fails for epistemic actions of the sort composed with epistemic states of the type specified by (9). Here, the situation is not that the epistemic action in question is redundant, but that it is *epistemically irrelevant*. The PC type epistemic action $(p \rightarrow q) \circ (p \rightarrow q)$ does *not* result in $p \rightarrow q$.

Contraction does not apply at all to PD type epistemic actions, on account of the epistemic states composing contracted epistemic actions carrying the same explicit informational payload (by definition), whilst the epistemic states composing PD type epistemic actions must be of different types (again by definition).

Consider Weakening.

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[15] Although both formulas are classically (and non-classically in certain logics) equivalent, recall that our epistemic agent α is not logically omniscient.

$$x \bullet y \sqsubseteq z \Longrightarrow x \sqsubseteq z \tag{Weakening}$$

Weakening fails outrightly for both PD and PC scenarios. To see this, consider the following PD scenario.

$$\begin{aligned} & x \Vdash p \rightarrow q \\ & y \Vdash p \\ & z \Vdash q \end{aligned} \tag{10}$$

Given the explicit epistemic states specified by (10), we have it that $x \bullet y \sqsubseteq z$. α's epistemic action combining α's explicit knowledge/belief of $p \rightarrow q$ and p is, along with the relevant epistemic states themselves (x and y) epistemically relevant to α's knowing explicitly that q. In other words, it is epistemically relevant to α being in the epistemic state z. However, $x \sqsubseteq z$ states that α's being in the epistemic state x, that is, their explicit knowledge that $p \rightarrow q$, is *non-contextually* epistemically relevant to their being in the epistemic state z, that is, their explicit knowledge that q. This it most certainly is not.

Now consider again the PC type epistemic actions specified by (6).

$$\begin{aligned} & x \Vdash p \rightarrow q \\ & y \Vdash q \rightarrow r \\ & z \Vdash p \rightarrow r \end{aligned} \tag{6}$$

Reasoning directly analogous to that entertained with respect to (10) demonstrates that Weakening fails for PC scenarios such as (6) also.

The failure of Weakening and Contraction for epistemic action scenarios is not entirely surprising insofar as brute considerations with regard to informational resources are concerned. The curious behaviour of Association and Commutation in our epistemic context is however, rather surprising indeed. There is, we should hope, a great deal more to say here with respect to structural rules, epistemic relevance, and epistemic actions.

4 Treating programs as data

But what of DC scenarios? When α is reasoning from a state x such that $x \Vdash p \rightarrow q$ (or any other piece of conditional information), it does not have to be the case that α's epistemic action is *an attempt* to combine this state with one carrying input-information, $y \Vdash p$ for example. Neither must it be the case that α is attempting to merge the information carried by this state with another state carrying information in conditional form as with scenario (6). This is just to say that α does not always have to treat dynamic information dynamically, so to speak. Instead, α may treat a program *as* data of a complex, non-atomic sort.

This will be the situation with many of α's epistemic actions. Consider those actions underpinning the merging of $p \rightarrow q$ with $(p \rightarrow q) \rightarrow r$ for example. In the context of this epistemic action, the dynamic information $p \rightarrow q$ is being treated by α as static data, input into the dynamic $(p \rightarrow q) \rightarrow r$. DC type epistemic actions build on this idea. Suppose that α is in the explicit epistemic state $x \Vdash ((p \rightarrow q) \wedge r) \rightarrow s$. Suppose also that α enters into two distinct sequences of reasoning, one of which brings α to state y such that $y \Vdash p \rightarrow q$, and another of which brings α to a state z such that $z \Vdash r$. In this case, for α to get to state w such that $w \Vdash (p \rightarrow q) \wedge r$, α will need to combine her states y and z *in such a way* that $y \bullet z \sqsubseteq w$, such that $w \Vdash p \rightarrow q, r$.

Importantly however, the "way" in which α combines y and z will be a way that treats the information carried by y as *data* to be combined with the data carried by z. This ensures that the result of the epistemic action $y \bullet z$ is w such that $w \Vdash p \rightarrow q, r$, as opposed to some failed attempt to *input* the information carried by z to the information carried by y. In other words, α knows that the epistemic action that she is executing with $y \bullet z$ is a DC scenario and not a PD one. A PD type epistemic action will in this case not be epistemically relevant to w at all, hence we would not have it that $y \bullet z \sqsubseteq w$.

The exact status of Boolean connectives $\wedge, \vee$, is something of a delicate matter. Although an agent may be reasoning with complex bodies of information which *contain* Boolean connectives, it is unlikely that the agent's epistemic states inherit all of the properties of these connectives. Consider the following under our epistemic state interpretation.

$$x \Vdash p \wedge q \text{ iff } x \Vdash p \text{ and } x \Vdash q. \tag{11}$$

$$x \Vdash p \vee q \text{ iff } x \Vdash p \text{ or } x \Vdash q. \tag{12}$$

In its left to right direction, (11) is true straightforwardly. If α knows/believes explicitly that $p \wedge q$, then α knows/believes explicitly that p and knows/believes explicitly that q. The right to left hand direction is slightly trickier however. (11) is true in its right to left hand direction, *given* the restricted case that it specifies. This is a consequence of it being the case that if α is in an explicit epistemic state x, which carries the information that p, and *that very same* explicit epistemic state x of α's carries the information that q, then x will carry $p \wedge q$. But this is not true of explicit epistemic states in general. It can be the case that α knows/believes explicitly that p, and that α knows/believes explicitly that q, without it being the case that α knows/believes explicitly that $p \wedge q$. Suppose that p and q are carried by explicit, but distinct epistemic states of α (x and y say). In this case there is no guarantee that α will have, or even so much as ever get to, some explicit epistemic state $z \Vdash p \wedge q$. For α to reach such a state z, α needs to execute a DC type epistemic action such that $x \bullet y \sqsubseteq z$.[16]

(12) is even less well behaved in a robustly epistemic context than is (11). In its right to left direction, (12) is well behaved epistemically. In its left to right direction

[16] For an investigation into the epistemic role of explicit conjunctions, especially with respect to the closure axiom and related modal-epistemic phenomena, see Sequoiah-Grayson (2013).

however, (12) fails for even the restricted case that it captures. It might well be true that α knows/believes explicitly that $p \vee q$, that is, α may be in state $x \Vdash p \vee q$, without it being the case that α knows/believes explicitly that p, or knows/believes explicitly that q. Consider an example from Dunn (2015), where you remember or believe that you left your keys either on the upstairs dresser, or on the basement workbench.[17] You could well be in the explicit epistemic state, without it being in that case that either of the disjuncts (considered independently) fall within the scope of that same epistemic state. Interestingly, there does not seem to be any obvious cognitive, or *a priori* executable epistemic action, DC type or otherwise, which would bring α to p or to q in this case. Rather, it would be an observation-type epistemic action.

This is to only touch on the issue of the DC type epistemic actions with regard to Boolean connectives. That there is more to say is obvious, but what to say is less so.

5 Conclusion

We have made a distinction between different types of omniscience, as well as different types of epistemic actions. Hopefully, a strong case has been made for a central role of such actions when it comes to *a priori* reasoning. Hopefully, a strong case has been made for the use of the structural rule architecture of relevance and related logics when it comes to modeling the properties of such actions for non-ideal, or non-logically omniscient agents also.

There has been a slow, but reassuringly steady interest in the applicability of relevant and related substructural logics to epistemic phenomena. See for example Majer and Pelis (2009) and their followup paper Bilkova et al. (2010). Relatedly, Sedlar (2012) makes explicit connections between universal modal operators and the ternary relation R. Given the role that such modal operators have played in traditional epistemic logic, the future along this route is promising. Relatedly, Sedlar (2014) and Roy and Hjortland (2014) explore epistemicised substructural modal logics to explore various epistemic phenomena of the epistemic action and epistemic update variety. There is hopefully much more to come.[18]

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[17] Although Dunn's remarks are not framed in explicitly epistemic terms, all of his examples concerning disjunction are epistemic/doxastic in nature. This is presumably no mere coincidence!

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