DYNAMIC NEGATION AND NEGATIVE INFORMATION

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Abstract

This essay proposes a procedural interpretation of negative information in terms of split negation as procedural prohibition. Information frames and models are introduced, with negation defined as the implication of bottom, **0**. A method for extracting the procedures prohibited by complex formulas is outlined, and the relationship between types of prohibited procedures is identified. Definitions of negation types in terms of the implication of **0** on an informational interpretation have been criticised. This criticism turns on the definitions creating a purportedly unnatural asymmetry between positive and negative information. It is demonstrated below that a strong asymmetry between positive and negative information is in fact the case. As such, an asymmetry between positive and negative information is natural, and something that we should want an informational interpretation of negation to preserve.

keywords: negative information, positive information, split negation, Wansing.

1 Introduction

Taking the procedural/dynamic turn in the study of information seriously means that we must make the transition from the study of bodies of information, to the study of the manipulations *of* such bodies of information. In this case, we will not be able to carry out the study of informational dynamics by restricting our attention to bodies of information, or even to the structure of the bodies of information, although this is an important component. We will also need to pay attention to the procedures via which such bodies of information are combined and developed, and processed.

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We may begin a study of informational dynamics by examining only *positive information*. That is, we may restrict our attention to the positive fragments of various logics used to underpin logics of information flow. For simple models of information—gain, a restriction to the study of positive information is justifiable on several counts, not the least of which is that it makes perfect sense to restrict ourselves to simpler cases, as even these may turn out to be surprisingly complicated. However, to do justice to the phenomena of information flow, any adequate theory of information processing will have to allow for the representation of both positive, and negative information — information canonically expressed using (sentential) negation. In this case, attention will not be restricted to the positive fragment of the various logics used to underpin logics of information flow.

This essay is an argument for a particular procedural interpretation of negative information. In particular, it is an argument for a procedural interpretation of split negation. A split negation pair $\langle \sim, \neg \rangle$ is definable in any non-commutative logic. As such, a procedural interpretation of split negation is adoptable in principle for any non-commutative logic, be this a non-commutative linear logic, or a variant of the Lambek calculus or whatever. Accordingly, we will be abstracting across non-commutative logics in general as opposed to looking in detail at any one non-commutative logic in particular. However, an information-processing application will be the general motivation. From a philosophical standpoint, the closest analogues are the non-commutative linear logics, albeit under procedural interpretations. Linear logics were developed in order to track resource-use: formulas are understood as resources, and in this case the number of times they occur becomes relevant. As such, the mark of linear logics in general is the rejection of contraction. However, if the formulas are taken to be concrete data, then the accessibility of these resources also becomes relevant. It is often the case that data have spatiotemporal locations, such as in the memories of agents or computers, and remote data will be less easy to access than adjacent data. In a more sensitive logic of resources then, it is not only the multiplicity of data, but also their order that is important. Spatiotemporal obstacles often need to be circumvented so that data may be accessed, hence commutation is inappropriate by virtue of its destroying the very ordering that we would like to preserve. In situations where actual information processing is being carried out, the arrangement of the data is crucial, as noted in Paoli (2002, 28-9). For recent work on non-commutative linear logics, see V.M. Abrusci, P. Ruet (2000), and for an explicitly procedural examination of commutation in the context of agent-based information processing, see Sequoiah-Grayson (2009).

Interpreting split negation is a known difficulty, for example see Dosen (1993, 20). For any negation type there will be more than one way of defining it. Given a definition, we then need to provide an interpretation of the resulting negation in terms commensurate with the intended application. In our case, the intended application is the area of dynamic information processing. Given the procedural aspect, we will define the negation of A in terms of A implying bottom ($\mathbf{0}$). This is commensurate with information processing due to the implication doing the work being analysed in procedural terms. Sans the procedural aspect, an interpretation of the negation of A in terms of A implying $\mathbf{0}$ goes back at least to Kripke (1965).

The first part of this essay develops and proposes a particular interpretation of the negation of A in terms of A implying $\mathbf{0}$ in information processing terms: In section 2, information frames and information models are introduced. It is here that we introduce the definition of split negation. In section 3, a procedural interpretation of split negation under the definition given in section 2 is proposed. Up to this point our exploration will have been conducted in purely model-theoretic terms. It is in section 3 that we touch on to proof-theoretical matters. This is essentially to check the procedural interpretation against a series of universally valid proof-theoretical properties of split negation. Put simply, the proposal is that we interpret the negation of A in terms of the ruling out of particular procedures, with these procedures being any procedure that involves combining the negation of A with A itself.

The second part of this essay responds to a fundamental philosophical objection to any analysis of negative information along the lines of A implying $\mathbf{0}$ - that it creates an unnatural asymmetry between positive and negative information. The asymmetry is conceded, but it is argued that it is entirely natural, and accurate. In section 4 we expound the objection in detail, firstly by translating it from its original intuitionistic logic form into the terms of our information frame from section 2, and secondly by identifying its crucial claims. In section 5, a defense of an asymmetry between positive and negative information is laid out via a demonstration that negative information is derivative upon positive information. In short, we should want an analysis of positive and negative information to reflect this asymmetry, rather than to bury it.

The first step is to introduce the notion of an information frame and model, so that we may specify our definition of split negation.

2 Information Frames and Models

Take a non-commutative information frame $\mathbf{F}$ $\langle S, \sqsubseteq, \bullet \rangle$ along with the three binary connectives $\otimes, \rightarrow$, and $\leftarrow$, and the constant $\mathbf{0}$. S is a set of information states x, y,... that may be inconsistent, incomplete, or both. The binary relation $\sqsubseteq$ is a partial order on S of informational development/inclusion. $\bullet$ is the binary composition operator on information states such that due to commutation failure we have it that $x \bullet y \neq y \bullet x$ for at least some $x, y, \ldots \in S$. $\otimes$ is (non-commutative) fusion. $\rightarrow$ and $\leftarrow$ are right and left implication respectively. $\mathbf{0}$ is bottom.[1][2] Making all of this clear is easier once we have a model.

[1]A notational note: $\mathbf{0}$ is commonly written as $\perp$. The difference in notation is to ensure that no confusion is made between bottom, and the perp relation of incompatibility Dunn (1993, 1994, 1996), written as $\perp$. In the recent literature on negation, $\perp$ is so often used for the perp relation that using it for bottom creates too great a risk for misunderstanding. Hence, we follow Girard (1987) in the use of $\mathbf{0}$ for bottom.

[2]Many non-commutative logics are also non-associative, such as the non-associative Lambek calculus among others. However, since nothing that follows depends on either the presence

A model $\mathbf{M} := \langle \mathbf{F}, \Vdash \rangle$ is an ordered pair $\mathbf{F}$ $\langle S, \sqsubseteq, \bullet \rangle$ and $\Vdash$ such that $\Vdash$ is an evaluation relation that holds between members of S and formulas constructed out of our binary connectives $\otimes, \rightarrow, \leftarrow$, as well as our constant $\mathbf{0}$. In what follows, we will often write $x, y, \ldots \in \mathbf{F}$ as shorthand for $x, y, \ldots \in S$ where $S \in F$. Where A is a propositional formula, and $x, y, z \in \mathbf{F}$, $\Vdash$ obeys the heredity condition:

$$\text{For all } A, \text{if } x \Vdash A \text{ and } x \sqsubseteq y, \text{then } y \Vdash A, \tag{1}$$

And also obeys the following conditions for each of our connectives:

$$x \Vdash A \otimes B \textit{ iff for some } y, z, \in \mathbf{F} \text{ s.t. } y \bullet z \sqsubseteq x, y \Vdash A \text{ and } z \Vdash B. \tag{2}$$

$$x \Vdash A \rightarrow B \textit{ iff for all } y, z \in \mathbf{F} \text{ s.t. } x \bullet y \sqsubseteq z, \text{if } y \Vdash A \text{ then } z \Vdash B. \tag{3}$$

$$x \Vdash A \leftarrow B \textit{ iff for all } y, z \in \mathbf{F} \text{ s.t. } y \bullet x \sqsubseteq z, \text{if } y \Vdash A \text{ then } z \Vdash B. \tag{4}$$

$$x \Vdash \mathbf{0} \text{ for no } x \in \mathbf{F}. \tag{5}$$

The evaluation relation $\Vdash$ may be understood in different ways, depending on the context of application. For example, if we were to be working with language frames and syntactically categorising particular alphabetical strings, we would understand $x \Vdash A$ to mean *string x is of type A*. We might instead consider a scientific research project with its various developmental phases. In this case the development relation $\sqsubseteq$ will order different states of a research project over time (with the idealisation that there is no information-loss). Here we would understand $x \Vdash A$ to mean that the proposition A is known at state x, and that this particular state of development in the project supports A. We will in fact return to this very idea in 5 below. For now however, we need something a little more general. Along with Mares (2009), we will understand $x \Vdash A$ to mean that *the state x carries the information that A*. Hence, we may also say that *x supports the information that A*. This is very similar to the familiar semantic entailment relation $\vDash$. The difference is that we want to allow for the information at x being incomplete and/or inconsistent. There are many applications where we might want to do this. Taking inconsistency as the running example, consider various states of an agent as the agent reasons deductively. In this case x may support A where A is *'p and not p'*, but this is different from x making A true, at least in the usual sense of "making true", as there is no possible way that the world could be such that A is true of it. One might wish to understand 'supports' as 'makes true' if one holds to a *dialethic paraconsistentism* whereby at least some contradictions are taken to be true. However, we will sidestep this particular debate and stay with the interpretation of 'supports' that takes it to be the subtler relative of 'makes true' in a manner aligned with Mares' informational interpretation.

The reader familiar with the ternary relation R of frame semantics will recognise (2)-(4) as the ternary conditions for $\otimes, \rightarrow$, and $\leftarrow$ respectively under an explicitly informational reading. R may be parsed in terms of the two binary

or absence of associativity, we should be able to safely ignore this issue for our purposes.

relations $\bullet$ and $\sqsubseteq$ and such that *Rxyz* comes out as $x \bullet y \sqsubseteq z$. How should we read formulas containing the binary composition relation $\bullet$? A common and traditional way of understanding binary composition is simply to take $x \bullet y$ as x together with y. In this case $\bullet$ will behave much the same as set union such that x together with y is no different to y together with x, and x together with itself is no different from x and so forth. However, we are not restricted to such a reading of $\bullet$. There is in general no canonical reading of *Rxyz*. That is to say that there is no canonical interpretation of the model theory. Although this is frustrating when one encounters the ternary relation for the first time, it is a key point with respect to the flexibility of ternary semantics. In our case, we have it that $\bullet$ is non-commutative, so $x \bullet y \neq y \bullet x$. In ternary terms, non-commutation comes out as $Rxyz \neq Ryxz$. Hence, simply interpreting $x \bullet y$ as x together with y will blur the very ordering that non-commutation is trying to preserve. We could stipulate that "x composed with y" differs from "y composed with x", however this is slightly strained and does not read straight off a casual use of 'composed'. A better way to keep this distinction robust is to read $x \bullet y$ as *x applied to y*. "Applying" is an order-sensitive notion, and one that fits comfortably with dynamic/procedural operations. So we can think of $x \bullet y$ as the composition of x with y, where this composition is order-sensitive, and we will mark this order-sensitivity by speaking of "application" instead of "composition".

Of course we are not merely concerned with syntactic constructions, as $x, y, z \in S$, and S is a set of information states. We are concerned with the application of the information in one state to the information in another. One way then, of reading *Rxyz*, is that *Rxyz* holds iff the result of applying the information in x to the information in y is contained in the information in z. This is precisely what $x \bullet y \sqsubseteq z$ tells us. Another way of putting this is to say that the information in z is a development of the information resulting from the application of the information in x to the information in y.

The role that *information application* plays here is not redundant, and neither is it *merely* to mark order-sensitivity. We are not simply concerned with ordered sequences of information states — something like an order—sensitive conjunction where we would have one piece of information, then another and then another etc. We are concerned with something much more subtle. We are concerned with the *interaction between* information states. This concern with interaction, or *process*, is precisely why it is that we are concerned with order-sensitivity in the first place. Order-sensitivity is in this sense a means to an end, with this end being the individuation of procedures of dynamic information processing. This sense of "applied" carries over in a natural way from the information states themselves, to the propositions supported by the information states. It is easiest to see this with an example.

Take fusion, and its frame conditions given in (2). (2) can be interpreted to state that an information state x carries the information resulting from the application of the information that A to the information that B if and only if x is itself a development of the application of the information in state y to the information in state z, where y carries the information that A and z carries the information that B. This is a little longwinded, and going in the right to left hand direction is more straightforward: for two states y and z that carry the information that A and that B respectively, the application of y to z will result in a new information state, x, such that x carries the information that results from the application of the information that A to the information that B. The analogous interpretations of the frame conditions for right and left implication ((3) and (4) respectively) unpack in a similar manner. What is happening here, is that the connectives, part of the language, are being interpreted using a relation between information states.

The fusion connective and the implication connectives are not independent; they form a family of sorts. Our fusion and implication connectives interrelate in the following manner:

$$A \otimes B \vdash C \textit{ iff } B \vdash A \rightarrow C \tag{6}$$

$$A \otimes B \vdash C \textit{ iff } A \vdash C \leftarrow B \tag{7}$$

In deductive information processing, we understand the premises as databases and the consequence relation '$\vdash$' as the information processing mechanism, a more brutally syntactic operation that the information carrying/supporting of $\Vdash$. In informational terms, we may read $A \vdash B$ as information of type B follows from information of type A, or the information in B follows from the information in A etc. We can think of typing as encoding, in which case we might also read $A \vdash B$ as the information encoded by B follows from the information encoded by A. (6) and (7) make sense.

Take (6), starting with the left-to-right-hand direction: If the information in C follows from the information resulting from the application of the information in A to the information in B, then from the information in B alone it follows that we have the information in C conditional on the information in A. The right-to-left-hand direction works out similarly: If from the information in B alone we can get the information in C conditional on the information in A, then we can get the information in C via the application of the information A to the information in B. Now take the left-to-right-hand direction of (7): If, again, the information in C follows from the information resulting from the application of the information in A to the information in B, then from the information in A alone it follows that we have the information in C, this time conditional on the information in B. The right-to-left-hand direction works on similarly here too: If from the information in A alone we can get the information in C conditional on the information in B, then we can get the information in C via the application of the information in A to the information in B. (6) and (7) are informational processing versions of the deduction theorem.

With regards to (5), no information, in any context whatsoever, is of type $\mathbf{0}$. There is nothing that we can do to get $\mathbf{0}$, and $\mathbf{0}$ is not supported by any information state in our frame $\mathbf{F}$.

Now we have the logical tools that we need in order to begin looking at negative information. We can define a split negation pair in terms of double implication:

$$\sim A := A \to \mathbf{0} \tag{8}$$

$$\neg A := \mathbf{0} \leftarrow A \tag{9}$$

In this case, the frame conditions for $\sim A$ and $\neg A$ are cashed out in explicit informational terms as follows:

$$x \Vdash \sim A\ [A \to \mathbf{0}] \textit{ iff } \text{for each } y, z \text{ s.t. } x \bullet y \sqsubseteq z, \text{ if } y \Vdash A \text{ then } z \Vdash \mathbf{0} \tag{10}$$

$$x \Vdash \neg A\ [\mathbf{0} \leftarrow A] \textit{ iff } \text{for each } y, z \text{ s.t. } y \bullet x \sqsubseteq z, \text{ if } y \Vdash A \text{ then } z \Vdash \mathbf{0} \tag{11}$$

The major points so far have been the informational translation of the ternary relation R, such that $Rxyz$ comes out as as $x \bullet y \sqsubseteq z$, and the definition of split negation in terms of double implication, such that $\sim A := A \to \mathbf{0}$ and $\neg A := \mathbf{0} \leftarrow A$. The definitional component here is important. Our double implication connectives $\to$ and $\leftarrow$ have their conditions given by R, albeit under an informational reading, in (3) and (4) respectively. This means that our split negation connectives $\sim$ and $\neg$ ultimately have their definitions in terms of the ternary relation also. With these technical details underlying split negation laid out, we may move on to matters more philosophical. The pressing issue ahead is to give an interpretation of our split negation, given its definition.

3 A Procedural Interpretation of Split Negation

How should we interpret $\sim A$ and $\neg A$ given their respective definitions, $A \to \mathbf{0}$ and $\mathbf{0} \leftarrow A$? The type of answer we give here will depend on the domain. For example, if we were working with actions, then we could interpret $\sim A$ as the type of action that cannot be followed by an action of type A, and we could interpret $\neg A$ as the type of action that cannot follow an action of type A etc. For any interpretation that we give to split negation, the interpretation has to be compatible with certain properties that hold universally for any split negation. The purpose of this section is to check the presently proposed procedural interpretation of split negation against these properties, which are listed as (12)-(18) below.

We are working with information. This is still fairly general though, and various informational applications will likely influence our choice of interpretation. By taking the procedural/dynamic turn and working with information *flow*, the *application* aspect at work in both fusion and the binary combination operator get taken very seriously. In this case, the suggestion is that we interpret $\sim A$ as the body of information that cannot be applied to bodies of information of type A, and that we interpret $\neg A$ as the body of information that cannot have bodies of information of type A applied to it. The interpretation is supported by the model theory; by the information states supporting $\sim A, \neg A$, and A. If x supports $\sim A$ and y supports A, x cannot be applied to y. Similarly, if x supports $\neg A$ and y supports A, then y cannot be applied to x. This is not because such an application will cause an explosion of information, but because it does not generate any information. This is not to say that such an attempted application is completely informationally bereft. We have a certain "meta—information" update concerning the informational redundancy of the procedure itself. We know via (5) that there is no information state in the network of possible information states that would support such an application. This is new information *about* the idle state that the process is in, rather than information generated by the process itself. The interpretation of split negation in terms of ruling out particular informational applications is not gerrymandered. It is directly supported by the frame conditions for $\sim A$ and $\neg A$.

To see this, note that the frame conditions in informational terms for $\sim A$ laid out in (10) above tell us that there is no information resulting from the application of x to y where $x \Vdash \sim A$ and $y \Vdash A$, since $x \bullet y \sqsubseteq z$ and $z \Vdash \mathbf{0}$ (and $z \Vdash \mathbf{0}$ nowhere). Suppose though that we were to attempt to apply $\sim A$ to A, in other words to attempt $\sim A \otimes A$. In informational terms, the frame conditions for fusion (2) tell us that an information state x will support $\sim A \otimes A$ iff for some information states y and z such that x is an informational development of the application of the information in y to the information in z, y supports $\sim A$ and z supports A. However, we know from our definition of $\sim A$ in terms of $A \rightarrow \mathbf{0}$, that there is no state x such that it supports the application of $\sim A$ to A, this is simply what (10) tells us.

Support for the ruling out conditions on $\neg A$ from the frame conditions for $\neg A$ works similarly, and involves only a directional change. The frame conditions for $\neg A$ laid out in (11) above tell us that there is no information resulting from the application of the information state y to the information state x where $y \Vdash A$ and $x \Vdash \neg A$ since $y \bullet x \sqsubseteq z$ and $z \Vdash \mathbf{0}$ (and $z \Vdash \mathbf{0}$ nowhere). If we were to attempt to apply$A$ to $\neg A$, in other words attempt $A \otimes \neg A$, then there would need to be an information state x that supported $A \otimes \neg A$, and this would be the case iff there were some information states y and z such that y supported A and z supported $\neg A$ and x was an informational development of the application of y to z. From our definition of $\neg A$ in terms of $\mathbf{0} \leftarrow A$ however, we know that there is no state x such that it supports the application of A to $\neg A$, this is marked out by (11).

Given the non-gerrymandered nature of the interpretation of split negation in terms of procedural prohibition, we should be able to give a natural interpretation of general proof-theoretic, hence information-processing properties of split negation in such terms. For any split negation, independently of which structural rules are present, the following (12)-(18) hold:

$$\frac{A \vdash B}{\sim B \vdash \sim A} \qquad (12)$$

(12) makes sense in terms of the ruling out of information processing procedures. Given a split negation, and given also that information of type B follows from information of type A, then ruling out the procedure $\sim A \otimes A$ follows from ruling out the procedure $\sim B \otimes B$. This is just to say that given that we can get information of type B from information of type A, then from the body of information that can never be applied to bodies of type B, we can get the body

of information that can never be applied to bodies of information of type A.

$$\frac{A \vdash B}{\neg B \vdash \neg A} \tag{13}$$

The reasoning with regards to (13) is directly analogous to that surrounding (12): Again given a split negation, and again given that information of type B follows from the information of type A, then ruling out the procedure $A \otimes \neg A$ follows from ruling out the procedure $B \otimes \neg B$. This is just to say that given that we can get information of type B from information of type A, then from the body of information that can never have bodies of type B applied to it, we can get the body of information that can never have bodies of information of type A applied to it. The reasoning surrounding (14) and(15) is slightly more involved than in (12) and (13).

$$\frac{A \vdash {\sim} B}{B \vdash \neg A} \tag{14}$$

$$\frac{B \vdash \neg A}{A \vdash {\sim} B} \tag{15}$$

We can take (14) and (15) together, getting the *split negation property*:

$$A \vdash {\sim} B \textit{ iff } B \vdash \neg A \tag{16}$$

Starting with the left—to—right—hand direction: If we can, on the basis of information of type A alone, get the body of information that can never be applied to bodies of information of type B, then on the basis of information of type B alone, we can get the body of information that can never have bodies of information of type A applied to it. The intermediate step is this: If we were to apply B to A (i.e., $B \otimes A$) then we would get nothing, viz. $\mathbf{0}$, since $B \otimes A \vdash \mathbf{0}$, since if $A \vdash {\sim} B$ then $B \otimes A \vdash \mathbf{0}$. As such, from information of type B alone we can get the body of information that can never have bodies of information of type A applied to it. This is straightforwardly laid out stepwise: from $A \vdash {\sim} B$, we get $A \vdash B \rightarrow \mathbf{0}$ via (8). From $A \vdash B \rightarrow \mathbf{0}$ we get $B \otimes A \vdash \mathbf{0}$ via (6), and from here we get $B \vdash \mathbf{0} \leftarrow A$ via (7). From $B \vdash \mathbf{0} \leftarrow A$ we get $B \vdash \neg A$ via (9). $\Box$

The right—to—left—hand direction is similar: If we can, on the basis of information of type B alone, get the body of information that can never have bodies of information of type A applied to it, then were to apply B to A (i.e., $B \otimes A$) then we would get nothing, viz. $\mathbf{0}$, since $B \otimes A \vdash \mathbf{0}$, since if $B \vdash \neg A$ then $B \otimes A \vdash \mathbf{0}$. As such, then from information of type A alone we can get the body of information that can never be applied to bodies of information of type B. Stepwise, the reverse of the left—to—right—hand direction: from $B \vdash \neg A$ we get $B \vdash \mathbf{0} \leftarrow A$ via (9), and from here we get $B \otimes A \vdash \mathbf{0}$ via (7). From $B \otimes A \vdash \mathbf{0}$ we get $A \vdash B \rightarrow \mathbf{0}$ via (6), and from here we get $A \vdash {\sim} B$ via (7). $\Box$

The double split negation introduction (DSNI) rules for our split negation operators (17) and (18) have interesting procedural consequences:

$$A \vdash \neg{\sim}A \tag{17}$$

$$A \vdash {\sim}\neg A \tag{18}$$

On the basis of information of type A alone, we can rule out the body of information that can never be applied to bodies of information of type A. This is just to say that we can rule out $A \rightarrow \mathbf{0}$. This is just what (17) states under a procedurally focused informational interpretation. Similarly, on the basis of information of type A alone, we can also rule out the body of information that can never have bodies of information of type A applied to it. In this case we are ruling out $\mathbf{0} \leftarrow A$. In this context, "ruling out" is a form of procedural prohibition. In terms of procedural prohibition, the DSNI rules tell us something interesting about the interaction between the different prohibitions. (17) tells us that given information of type A, we can never apply A to the body of information that can never be applied to bodies of information of type A. To see this, note that from $A \vdash \neg{\sim}A$ we get $A \vdash \mathbf{0} \leftarrow {\sim}A$ via (9), and that from here we can get $A \otimes {\sim}A \vdash \mathbf{0}$. Similarly, (18) tells us that given information of type A, we can never apply to A the body of information that can never have bodies of information of type A applied to it. From $A \vdash {\sim}\neg A$ we get $A \vdash \neg A \rightarrow \mathbf{0}$ via (8), and from here we get $\neg A \otimes A \vdash \mathbf{0}$ via (7). These interactions between our procedural prohibitions carry over to more complex procedures.

By the residual conditions laid out in (6) and (7) above, along with the definition of split negation in terms of arrows, we already know something of the relationship between intensional structure and negation. Since our split negation is defined intensionally from the start, this is as we would expect. We can specify some further relationships between our negations connectives and intensional structure under a procedural interpretation. If the interpretation of split negation in terms of procedural prohibition is to truly get off the ground, it must be able to handle more complex formulas. The first task is to demonstrate that we may sensibly read off an interpretation of such complex formulas in terms of procedural prohibition, in much the same manner as we have done so far. The second task is to demonstrate that we may extract the specific procedures prohibited from complex formulas containing split negations. We have already seen a proto—version of this process with the expansion of the DSNI cases above. Take the following:

$$A \rightarrow \neg B \vdash B \rightarrow {\sim}A \tag{19}$$

(19) makes procedural sense. Suppose that conditionally on information of type A, we have the body of information that cannot have bodies of information of type B applied to it. It follows from this that conditionally on information of type B, that we have the body of information that can never be applied to bodies of information of type A. In other words, given that conditionally on information of type A, that we can rule out the procedure $B \otimes \neg B$, it follows that conditionally on information of type B, that we can rule out the procedure

$\sim A \otimes A$. Note that "conditionally" comes procedurally preloaded in the sense that (19) prohibits a more complex specific procedure. The complex procedure that (19) prohibits is the following:

$$A \otimes (B \otimes (A \rightarrow \neg B)) \tag{20}$$

From (19) and the right—to—left hand direction of (6), we get $B \otimes (A \rightarrow \neg B) \vdash \sim A$, validating the surface claim of (19). Via (8), this is just to say that we get $B \otimes (A \rightarrow \neg B) \vdash A \rightarrow \mathbf{0}$. By a second application of the right—to—left hand direction of (6), we get $A \otimes (B \otimes (A \rightarrow \neg B)) \vdash \mathbf{0}$. □

With (20), we have our first instance of a successfully extracted prohibited procedure from a sequent composed of complex formulas containing split negation. Note a pattern that we will see repeated in further examples. Note that both A and B get applied to the left of the formula on the left hand side of the original sequent in (20). This is because both A and B occur on the left hand side of the arrows on the right hand side of (19) (the expanded form of the right hand side of (19) is $B \rightarrow (A \rightarrow \mathbf{0})$). When we transfer bodies of conditionalised information from the right hand side of a sequent to the left, they "remember" which side of the residual they occurred on. Taking X to stand for the structured procedure $B \otimes (A \rightarrow \neg B)$, then what we learn from the prohibited procedure extraction from (19) is that if from X we can get the body of information that can never be applied to bodies of information of type A, it is also the case that we can never apply A to X. Generalising, if it is the case that from processing on X we can get the body of information of *type* $\sim A$, then X itself must be of *type* $\neg A$. This relationship will become clearer with the following cases.

$$A \rightarrow \sim B \vdash B \rightarrow \neg A \tag{21}$$

(21) also makes procedural sense. This time, given that conditionally on information of type A we can rule out the procedure $\sim B \otimes B$, it follows that conditionally on information of type B, we can rule out the procedure $A \otimes \neg A$. The prohibited procedure underpinning (21) is:

$$(B \otimes (A \rightarrow \sim B)) \otimes A \tag{22}$$

Via the right—to—left hand direction of (6) again, we get $B \otimes (A \rightarrow \sim B) \vdash \neg A$, validating the surface claim of (21). Via (9), this is just to say that we get $B \otimes (A \rightarrow \sim B) \vdash \mathbf{0} \leftarrow A$. Via an application of the right—to—left hand direction of (7), we get $(B \otimes (A \rightarrow \sim B)) \otimes A \vdash \mathbf{0}$. □

Note that with (22), B is applied to the left of the formula on the left hand side of the sequent in (21) just as it was in (20) above. A however, is this time applied to the right. This is because whilst B occurs of the left hand side of its arrow on the right hand side of (21), A occurs on the right hand side of its arrow via (9) (i.e., the expanded form of the right hand side of (21) is $B \rightarrow (\mathbf{0} \leftarrow A)$). This time taking X to stand for the structured procedure $B \otimes (A \rightarrow \sim B)$, then what we learn from the prohibited procedure extraction from (21) is that is from X we can we can get the body of information that can never have bodies of information of type A applied to it, then it is also the case that we cannot apply X to A. In other words, if it is the case that from processing on X we can get the body of information of type $\neg A$, then X itself must be of type $\sim A$.

Consider a directional change, so that we have B on the right hand side of the conditionalised procedure on the right hand side of the sequent. In this case we have, respectively:

$$A \rightarrow \neg B \vdash \sim A \leftarrow B \tag{23}$$

$$A \rightarrow \sim B \vdash \neg A \leftarrow B \tag{24}$$

The prohibited procedure underpinning (23) is the following:

$$A \otimes ((A \rightarrow \neg B) \otimes B) \tag{25}$$

Begin by applying the right—to—left hand direction of (7) to (23) so as to get $(A \rightarrow \neg B) \otimes B \vdash \sim A$, validating the surface claim of (23). Via (8) we have $(A \rightarrow \neg B) \otimes B \vdash A \rightarrow \mathbf{0}$. Via the right—to—left hand direction of (6), we get $A \otimes ((A \rightarrow \neg B) \otimes B) \vdash \mathbf{0}$. □

With (25), B is applied to the right of the formula on the left hand side of (23), whilst A is applied to the left. This is because A occurs on the left hand side of its arrow on the right hand side of (23), whilst B occus on the right hand side of *its* arrow (i.e., the expanded form of the right hand side of (23) is $(A \rightarrow \mathbf{0}) \leftarrow B$). Although the structured procedure giving us the type $\sim A$ from (23) is different from that of (19) (since B *is on the right*, it is type—identical insofar as prohibited procedure is concerned, namely it is of type $\neg A$.

The prohibited procedure underpinning (24) is the following:

$$((A \rightarrow \sim B) \otimes B) \otimes A \tag{26}$$

We begin by applying the right—to—left hand direction of (7) to (24) so as to get $C \vdash \neg A$, validating the surface claim of (24). Via (9) we have $(A \rightarrow \sim B) \otimes B \vdash \mathbf{0} \leftarrow A$. Via another application of (7), we get $((A \rightarrow \sim B) \otimes B) \otimes A \vdash \mathbf{0}$. □

With (26), both A and B are applied to the right of the formula occurring on the left hand side of the sequent in (26). The reasons for this should be obvious enough by now (the expanded form of the right hand side of (24) is $(\mathbf{0} \leftarrow A) \leftarrow B$). Similarly to the reasoning above, since we get the body of information of type $\neg A$ from the structured procedure $B \otimes (A \rightarrow \sim B)$, $B \otimes (A \rightarrow \sim B)$ is of type $\sim A$. What is perhaps less obvious is the moral: when we are searching for the prohibited procedure underpinning sequents of conditionalised bodies of information, we do not need to go through the inferential steps (which might well be lengthly). We simply apply the relevant informational antecedents to the left and/or right hand side(s) of the formula on the left hand side of the original sequent until the only thing left on the right hand side is $\mathbf{0}$, then we have got it. We also know the relationship between the types of procedural prohibition:

$$\textbf{type} \sim A \dashv\vdash \textbf{type}\ \neg A \tag{27}$$

The proposal for a procedural interpretation of split negation has been that we interpret $\sim A$ (that is $A \rightarrow \mathbf{0}$) as the body of information that cannot be applied to bodies of information of type A, and that we interpret $\neg A$ (that is $\mathbf{0} \leftarrow A$) as the body of information that cannot have bodies of information of type A applied to it. We have seen that this interpretation of split negation is entirely natural once we translate the ternary relation R into its informational form, in which case the interpretation is directly supported by the frame conditions for $\sim A$ and $\neg A$, (10) and (11) respectively. We have also seen that the interpretation is compatible with the universally valid split negation properties (12)—(18), as well as those properties of split negations and complex formulas (19)—(27). This however, is only half the job done. There remains a fundamental objection that goes to the heart of any informational interpretation of negation (split or otherwise) that that defines the negation of A in terms of A implying $\mathbf{0}$. The basic thrust of this objection is that positive and negative information should be awarded equal status, but that the definition of the negation of A in terms of A implying $\mathbf{0}$ affords positive information an unfair advantage over its negative sibling. The purpose of the following section is to elucidate this objection in more detail.

4 An Objection to Positive Discrimination

There is a pressing objection to defining negation in terms of implying $\mathbf{0}$. In this section, we will lay out the details of just what exactly the objection is, leaving the argument *for* the objection until the following section. The relevant debate is played out in intuitionistic logic. As such, we will state it in intuitionistic terms before translating it into our expressively richer ternary terms. Given that intuitionistic systems are commutative, our split negation collapses into a single negation, which we will write as '$-$'. Of course, there are more differences that this between intuitionistic systems and the non-commutative systems, be they non-commutative linear logics, or variants of the Lambek calculi or whatever. However, we can safely abstract away from them for our purposes. Similarly to our definition of split negation above, intuitionistic negation is defined in terms of implication. In this case however, the implication is intuitionistic implication, which we will write as '$\sqsupset$'. The frame conditions for $\sqsupset$ are as follows:

$$x \Vdash A \sqsupset B \textit{ iff } \text{for all } y \in \mathbf{F}, \text{ s.t. } x \sqsubseteq y, \text{ if } y \Vdash A \text{ then } y \Vdash B \tag{28}$$

Hence:

$$-A := A \sqsupset \mathbf{0} \tag{29}$$

Hence the frame condition for intuitionistic negation is:

$$x \Vdash -A \, [A \sqsupset \mathbf{0}] \textit{ iff } \text{for all } y \in \mathbf{F}, \text{ s.t. } x \sqsubseteq y, \text{ if } y \Vdash A \text{ then } y \Vdash \mathbf{0} \tag{30}$$

(30) simply states that if x caries the information that $-A$, then there no state y such that y is an informational development of x where y carries the information that A. With this much laid down, we can expound that objection to

intuitionistic negation. The objection works like this: Assume that we want to treat representations of positive and negative information on an equal footing. Then the definition of $-A$ in terms of $A \sqsupset \mathbf{0}$ throws up an unwanted asymmetry. In an information model $-A$ holds at $x \in \mathbf{F}$ iff A does not hold at any $y \in \mathbf{F}$ such that $x \sqsubseteq$ y. Whilst the verification of A at $x \in \mathbf{F}$ only involves checking x, verifying $-A$ at $x \in \mathbf{F}$ involves checking all $y \in \mathbf{F}$ such that $x \sqsubseteq y$. According to Gurevich (1977) and Wansing (1993), this asymmetry means that intuitionistic logic does not provide an adequate treatment of negative information, since, unlike the verification of A, there is no way of verifying $-A$ "on the spot" so to speak.

A version of this objection is translated straightforwardly into split negation terms: $\sim A$ holds at $x \in \mathbf{F}$ iff there is no $z \in \mathbf{F}$ such that it is an informational development of the application of x to any $y \in \mathbf{F}$ such that y verifies A. Hence, verifying $\sim A$ at $x \in \mathbf{F}$ involves checking all $y, z \in \mathbf{F}$ such that $x \bullet y \sqsubseteq z$. We have an analogous situation with $\neg A$: $\neg A$ holds at $x \in \mathbf{F}$ *iff* there is no $z \in \mathbf{F}$ such that it is an informational development of the application of any $y \in \mathbf{F}$ such that y verifies A. Hence, verifying $\neg A$ at $x \in \mathbf{F}$ involves checking all $y, z \in \mathbf{F}$ such that $y \bullet x \sqsubseteq z$. Since it is that case that checking $x \Vdash A$ involves checking x only, we have the same asymmetry between positive and negative information present in our definition of split negation as obtains in the parallel definition of intuitionistic negation.

How should we respond to this objection with regards to the procedural interpretation of split negation? Before we can answer this question, we need to get clearer on what exactly it is that Gurevich/Wansing are arguing. So far we have the following (translating from intuitionistic to ternary terminology):

(a) Any *adequate* theory of information processing will allow for representing both positive and negative information.

(b) The definition of split negation in a model $\mathbf{M}$ has the result that positive and negative information are treated asymmetrically; A may be verified "on the spot", whilst $\sim A$ and $\neg A$ may not.

(c) Therefore, any theory of informaiton processing based upon $\mathbf{M}$ will not be an adequate theory of information processing.

As things stand, (c) does not follow from (a) and (b), as a theory of information processing based upon $\mathbf{M}$ does allow for the representation of both positive and negative information. There is nothing in (a) stating that such representations need to be in symmetry. Obviously there is a missing premise, and there are two that fit the bill:

(a′) The representation of positive information must be in symmetry with the representation of negative information in order for a theory of information processing to be adequate.

(a″) Bodies of either positive or negative information must be directly verifiable in order for a theory of information processing to be adequate.

(a$''$) is the stronger claim, since if we have satisfied it then we have *ipso facto* satisfied (a$'$). We could, in principle at least, have a verification condition on A that was just as "off the spot" as are the present verification conditions on $\sim A$ and $\neg A$, in which case symmetry would be satisfied. Similarly, refuting (a$'$) refutes (a$''$), but not vice-versa. Which of either (a$'$) and (a$''$) are the intended premise? Gurevich seems to be arguing for the weaker (a$'$) when he states that"[i]n many cases the falsehood of a simple scientific sentence can be ascertained as directly (or indirectly) as its truth" Gurevich (1977, 49), (my emphasis). Wansing sometimes seems to be arguing for (a$''$) when he states "Gurevich's remark amounts to the complaint that there is no possibility of direct falsification of [A] on the spot", Wansing (1993, 14). However, other comments such as "...the idea of taking negative information seriously and putting it on par with positive information leads Gurevich to intuitionistic logic with strong negation...", Wansing (1993, 14), are much more in line with (a$'$). We take it then, that the more flexible (a$'$) is the missing premise. In this case, the full form of the argument is (a), (a$'$), (b), therefore (c). This argument is valid. Now that we have reconstructed it, we may formulate a response on behalf of the procedural interpretation of split negation. Our task is to deny the argument's soundness by rejecting (a$'$). This is the task undertaken in the following section.

5 A Defense of Positive Discrimination

We have seen that there is an objection to be raised against any informational interpretation of negation (split or otherwise) that defines the negation of A in terms of A implying $\mathbf{0}$. The central component of this objection is that there should be no asymmetry between representations of positive and negative information - they should be afforded equal status, viz. (a$'$) above. Any informational interpretation of negations based upon a definition of the negation of A as A implying $\mathbf{0}$ will violate this condition, since the verification of A at x will involve checking only x, whilst the verification of the negation of A at x will involve checking other states along with x. The task of this section is to expose and reject the reasons for supposing that positive and negative information should be afforded an equal status in the first place. With this done, we will have defended (8) and (9), hence vindicated the definition of $\sim A$ as $A \rightarrow \mathbf{0}$ and the definition of $\neg A$ as $\mathbf{0} \leftarrow A$.

What is the motivation for accepting (a$'$) in the first place? Recall again Gurevich's remark that "[i]n many cases the falsehood of a simple scientific sentence can be ascertained as directly (or indirectly) as its truth" (*op. cit.*). The reference to scientific reasoning is not happenstance. Gurevich is working in the spirit of Grzegorczyk (1964) whereby a Kripke model $\mathbf{K} := \langle S, \sqsubseteq, d, \Vdash \rangle$ of intuitionistic predicate logic is "interpreted as a scheme for scientific research" Gurevich (1977). Put briefly, the basic idea is this: In $\mathbf{K}$, $x, y, \ldots \in S$ are taken to be stages of scientific research, $\sqsubseteq$ is taken to be the partially ordered precedence relation on $S, d(x)$ is the set of objects involved in research at stage x, and for any *atomic* formula A, $x \Vdash A$ is the outcome of experiment. There are more details than this of course, but these are enough to get us going. The relevant point is that Grzegorczyk only allows atomic formulas to be experimental outcomes. On his view, compound sentences are not outcomes of experiment. Grzegorczyk states that compound sentences "...arise from reasoning. This concerns also negations: we see that the lemon is yellow, we do not see that it is not blue" Grzegorczyk (1964, 596). Grzegorczyk's condition for intuitionistic negation is as (29) above. Hence, Grzegorczyk is rejecting anything along the lines of (a′) with respect to scientific reasoning, taking the difference in positive and negative information to be a natural consequence of only atomic sentences being outcomes of experiment.

Gurevich's position is a reaction against Grzegorczyk. Gurevich's example of scientific sentences that have their falsehood directly ascertained is "The solution is acid" with regards to a litmus paper test. However, falsification can happen just as well as a result of positive information as negative information. In fact, contra Gurevich, the litmus paper example is an instance of just this.

Suppose that we are testing for acid, and that the paper remains blue (blue litmus paper turns red in an acid, red litmus paper turns blue in a base). In this case, we have falsified "The solution is acid". But on the basis of what? The falsification proceeds via the positive information that the paper is still blue. The negative information concerning the falsification of "The solution is acid" is derivative upon the positive information concerning the blueness of the paper. Even in the restricted context of scientific reasoning, it is certainly not straightforward that negative information should be granted the status of an "informational primitive", on par with positive information. Adapting Grzegorczyk's point from the paragraph above, we see that the paper is blue, we do not see that it is not red. We do not "see that it is not red" any more than we "see that it is not a cat". We acquire the negative information *it is not the case that the solution is acid* on the basis of the positive information *the paper is blue* that is incompatible with the positive information *the solution is acid*. We ascertain that it is not red because we see that it is blue.

The important point to emphasise is this: Wansing is arguing for a *symmetrical* treatment of positive and negative information. The motivation for this is the supposed direct ascertainment of the falsehood of sentences in scientific practice. However, even if a certain number of such falsehoods *were* demonstrably ascertained directly, this would be insufficient to demonstrate a *symmetry* between positive and negative information, even if we were to restrict our attention to contests of experimental reasoning. If we were to be picky, we might demand that all of the instances of negative information be ascertained directly for a demonstration of symmetry strictly conceived. However, we do not need to go this far. All that we need to do is to say that a significant amount of negative information is indirectly acquired, with the onus to demonstrate otherwise resting on those who wish to so demonstrate.

Even in the canonical context of scientific reasoning, negative information appears to be derivative upon positive information. This fact is in line with the formulation of intuitionistic negation given by (29), and our definition of split negation in terms of the ternary relation. There is an important sense in which this is by the by however. (a′) is a claim about theories of information processing in general. It is not restricted to theories of information processing with respect to models of stages of scientific research. So, even if it was straightforwardly the case that we "ascertain the falsehood of a simple scientific sentence as directly as its truth" (and it is almost certainly not the case at all, not even non—straightforwardly), it would remain open to us to make the case that in the general context of a logic of information, a non-symmetric treatment of representations of positive versus negative information is adequacy enough (and so satisfying (a)).

Further asymmetries are not difficult to come by. Take it that facts ground information.[3] In this case, an asymmetry between positive facts and negative facts will carry over into an asymmetry between positive and negative information. There are two ways that this can go. The first is to follow those who deny the existence of negative facts outright. In this case, positive truths have have facts as truthmakers, whereas negative truths have no truthmakers at all (they simply lack truth), or at least are not made true by anything existent. The second (and more subtle approach) is to hold that positive information is grounded in particular facts, whereas negative information is not. For example, the information that there is a car parked in my driveway is grounded by the car parked in my driveway. The negative information that there is no elephant in my driveway is not grounded by a particular fact, it is grounded by my driveway as a whole. Either way, we have an asymmetry.

We have a natural asymmetry with the very context of procedural interpretation of split negation that we are considering. In a system of procedural information processing, it is completely natural to interpret $\sim/\neg A$ as the ruling out of a procedure. The procedure ruled out by $\sim/\neg A$ is just any procedure that involves combining $\sim/\neg A$ with A itself. In summary, insofar as general concerns regarding the indirectness of such a definition of negation in informational terms is concerned, it is worth considering the observation that an interpretation of negation in terms of ruling something out is about as direct as we could want. "Ruling out" is a direct notion. But to rule something out requires a "universal checking".[4] This kind of *in*directness is required in order that something be truly ruled out. To put this another way, how can we rule something out without first considering all the possible cases?

6 Conclusion

We have seen how it is that we may reconstruct the ternary relation of frame semantics, $Rxyz$ in explicitly dynamic informational terms, as $x \bullet y \sqsubseteq z$. This

[3]A more likely state of affairs if we understand information to be veridical. This is the position argued for by Dretske (1981). For related arguments for the veridicality of particular information types, see Floridi (2004) and Sequoiah-Grayson (2007).

[4]Many thanks to Catarina Dutilh for getting me to think about "ruling out" in universal terms.

dynamic informational reconstruction carries over to any connective defined in terms of the ternary relation, allowing us to give explicitly procedural accounts of double implication and fusion. Since we have used double implication to define a split negation, $\sim A := A \rightarrow \mathbf{0}$, and $\neg A := \mathbf{0} \leftarrow A$, we have a procedural definition of split negation.

Given the definition of split negation in these dynamic informational terms, we have been able to "read off" a natural procedural interpretation of split negation. This interpretation has been shown to be compatible with the universally valid properties of a split negation.

The central philosophical rejection of the definition of split negation in terms of the implication of bottom is that it creates an unnatural asymmetry between positive and negative information. The suggestion is that this asymmetry is but a natural and accurate representation of the facts concerning the behaviour of positive and negative information.[5]

References

V.M. Abrusci, P. Ruet, 2000: Non-commutative logic, I: the multiplicative fragment, Ann. Pure Appl. Logic (2000) 29—64.

Dosen, K. 1993, A Historical Introduction to Substructural Logics, in P. Schroeder-Heister and K. Dosen (eds.), *Substructural Logics, Studies in Logic and Computation no. 2*, Oxford Science Publications, Clarendon Press, Oxford: 1—30.

Dretske, F. (1981), *Knowledge and the Flow of Information*, Cambridge, MA: MIT Press, rep. Stanford: CSLI, 1999.

Dunn, J. M. (1993), Partial Gaggles Applied to Logics with Restricted Structural Rules, in P. Schroeder-Heister and K. Dosen (eds.) *Substructural Logics, Studies in Logic and Computation no. 2*, Oxford Science Publications, Clarendon Press, Oxford: 63—108.

Dunn, J. M. (1994): Star and Perp: Two Treatments of Negation, in J. E. Tomberlin (ed.), *Philosophical Perspectives*, Vol. 7, 331—57.

Dunn, J. M. (1996): Generalised Ortho Negation, in H. Wansing (ed.): *Negation: A Notion in Focus*, Walter de Gruyter, Berlin, 3—26.

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Floridi, L. (2004): Outline of a Theory of Strongly Semantic Information, *Minds and Machines*, 14, 192-221.

Girard, J-Y. (1987): Linear Logic, *Theoretical Computer Science*, 50, 1—101.

Grzegorczyk, A. (1964): A Philosophically Plausible Interpretation of Intuitionistic Logic, *Indagnationes Mathematicae*, 26, 596-601.

Gurevich, Y. (1977): Intuitionistic Logic with Strong Negation, *Studia Logica*, 36, 49—59.

Kripke, S. A. (1965): Semantical Analysis of Intuitionistic Logic I, in J. Crossley and M. Dummett (eds.), *Formal Systems and Recursive Functions*, North-Holland, Amsterdam, 92—129.

Mares, E. (2009): General Information in Relevant Logic, forthcoming in Synthese, section: Knowledge, Rationality, and Action, L. Floridi and S. Sequoiah-Grayson (eds.): *The Philosophy of Information and Logic, Synthese, KRA*, Proceedings of PIL-07, The First Workshop on the Philosophy of Information and Logic, University of Oxford, November 3—4, 2007.

Paoli, F. (2002): *Substructural Logics: A Primer*, Springer.

Sequoiah-Grayson, S. (2007): The Metaphilosophy of Information, *Minds and Machines*, 17, 331—344.

Sequoiah-Grayson, S. (2009): A Positive Information Logic for Inferential Information, forthcoming in *Synthese*, section: *Knowledge, Rationality, and Action*, L. Floridi and S. Sequoiah-Grayson (eds.): *The Philosophy of Information and Logic, Synthese, KRA*, Proceedings of PIL—07, The First Workshop on the Philosophy of Information and Logic, University of Oxford, November 3—4, 2007.

Wansing, H. (1991): The Logic of Information Structures, *Lecture Notes in Artificial Intelligence no. 681, Subseries of Lecture Notes in Computer Science*, Berlin, Springer-Verlag.