A logic of affordances

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Abstract: I motivate and develop a formal account of inference rules as specifications of environmental affordance-types such that they afford us opportunities for successful epistemic action across abstract environments. The formal account is given by a weak substructural logic, motivated by a proceeding philosophical discussion.

Keywords: affordances, inference rules, omniscience, substructural logic, types

1 Introduction

Actions are things that we do, perform, or execute. What we are able to do successfully will depend typically on two things. Firstly, what we are able to do successfully will depend on our skillset or acumen. Secondly, what we are able to do successfully will depend on environmental affordances - those opportunities for action that our environment affords us, (Gibson, 1966), (Rolands, 1997), (Bermudez, 1998).

For example, suppose that the action in question is my playing Bach's *Cello Suite no. 1* on the cello. For me to be able to perform this action successfully at a given point in time, then instances of the two conditions above must obtain. Firstly, I must know how to play Cello Suite no. 1 on cello in the first place, and secondly, there needs to be an actual cello around. If either of these conditions are not met, then I shall not be able to perform the action in question successfully.

The example above (as with examples of affordances traditionally), targets the physical environment. The proposal for which I shall argue here is that we may think of certain abstract environments, namely those populated by logic-mathematical entities and structures, as *bona fide* environments such

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that they may contain affordances in a manner that is not totally disanalogous to that which is described above. In particular, I want to motivate and develop a formal account of *inference rules as specifications of environmental affordances such that they afford us opportunities for successful epistemic action across abstract environments*.

2 Metaphysical prerequisites - platonism

We assume a robust logico-mathematical platonism, (Williamson, 2002). The abstract environment therein is populated by logically existent objects that are necessarily non-concrete hence abstract. The truth-makers for statements referring to them are mind-independent, but this is not to say that there is no role for the mind play when it comes to navigating or perceiving this environment. Indeed, perception of the logico-mathematical environment (LME hereafter) appears to be uniquely mental insofar as it is via mental actions, acts of the mind, by which we traverse it.

Like the physical environment, the LME affords us multiple opportunities for action. These actions will be *epistemic actions*. We will explore epistemic actions in some detail in section 5 below. For now, we can understand an epistemic action to be any action that is precipitated by a desire to relieve an epistemic deficit. Inferences are canonical examples. Again like the physical environment, what we are able to do successfully in the LME will depend typically on two things. Firstly, what we are able to do successfully will depend on our skillset or acumen. Secondly, what we are able to do successfully will depend on environmental affordances - those opportunities for action that our environment affords us.

In the LME, it is our logical acumen, that is our relevant mental skillset, that will make the difference. Similarly, the opportunities for action that this environment will afford us will depend on the abstract artefacts that populate the proper part of the LME to which we are attending. It is a mistake to think that attending to one part of the LME is to attend to all of it. In a slogan:

Slogan 1 a priori *knowability is not knowability for free.*[2]

Whatever the informational architecture of the LME might be, it is what it is necessarily. Hence our actions cannot change it. Rather, our actions in the LME are actions of the mind or understanding such that they may, all things

[2]By way of a suitably dramatic example, consider Frege's initial failure to notice the inconsistency resulting from his unrestricted comprehension axiom (Basic Law V).

going well, take us to new logico-mathematical facts. The perception of such logico-mathematical facts will involve new understandings, or new mental states, since it is through our faculties of understanding that such facts are perceived. In another slogan:

Slogan 2 *Our actions in the LME cannot change that environment, but they can change us.*

In the following section, we build on our understanding of the LME and develop the proposal that inference rules are specifications of environmental affordances such that they afford us opportunities for successful epistemic action across abstract environments in more detail.

3 Development

We will begin this development with a third slogan:

Slogan 3 *Inference rules may be understood as specifications of affordances across the LME, with respect to target propositions.*

Take as an example disjunction - $\vee$, and consider the rule of *disjunctive syllogism* (DS). What DS tells us is that if we “have” $\phi \vee \psi$, then if we “have” $\neg\phi$ then we “may get” ψ, and if we “have” $\neg\psi$ then we “may get” ϕ.

The scare quotes are to flag the following. By “have $\phi \vee \psi$”, we mean that we are attending to the proper part of the LME that is $\phi \vee \psi$. By “attending to” we mean that $\phi \vee \psi$ is the explicit target proposition of some propositional attitude or other. Such an attitude may be entirely non-committal, such as *entertaining*, *supposing*, or *assuming*, or involve non-factive assent, such as *belief*, or factive assent, such as *knowledge*. By “may get ψ" if we have $\neg\phi$, we mean a bundle of two things. The first thing is that DS is an inference rule with which we are competent to a reasonable degree. By reasonable degree we do not mean the exclusion of error, any more than we would mean the exclusion of error by stating that we can play Bach’s *Cello Suite no. 1*. The second thing is that it does not follow from it being true that we may get ψ that it is true that we *have gotten* ψ. In order to get ψ from $\phi \vee \psi$ and $\neg\phi$ with DS, we need to perform the inference in question, to execute the *epistemic action*. In a fourth slogan:

Slogan 4 *When performing epistemic actions, we avail ourselves of the relevant environmental affordance.*

Again, epistemic actions are those actions whose purpose is to relieve us of some epistemic deficit or other. Just as observations are canonical epistemic actions in the physical environment, inferences are canonical epistemic actions in the logico-mathematical one.

Environmental artefacts present multiple affordances typically, and this is no less true for the abstract artefacts populating the LME than it is for the concrete objects populating the physical one. What DS tells us is that a disjunction presents to us a pair of affordances. In more detail, what DS tells us in particular, is that the artefact $\phi \vee \psi$ is the type of thing that might afford us ϕ on the one hand, or ψ on the other, but it does not do so unconditionally! Rather, DS tells us that $\phi \vee \psi$ is of the type $\neg\phi \rightarrow \psi$, and the type $\neg\psi \rightarrow \phi$. Using standard type-theoretic notation, we may say that DS tells us that $\phi \vee \psi : \neg\phi \rightarrow \psi$ and $\phi \vee \psi : \neg\psi \rightarrow \phi$. Moreover, DS will specify affordance types when the target propositions are negations. Here too, the affordances are not unconditional. Rather, DS tells us that $\neg\phi : (\phi \vee \psi) \rightarrow \psi$, and $\neg\psi : (\phi \vee \psi) \rightarrow \phi$ and so on. Hence the following slight adjustment to Slogan 3 above:

Slogan 5 *Inference rules specify affordance* types *in the LME, with respect to target propositions.*

It is important to recognise that the specification of an affordance type is not the same thing as our having availed ourselves of the affordance, or *actualised* it. The affordances above are conditional on our targeting the antecedent, or attending to the proper part of the LME populated by the antecedent, and *then* combining it with the conditional type in the appropriate manner. Just what it is to which “combine in appropriate manner" amounts will depend on the properties (for example, *identity and logical form*) of the target propositions and affordance types in question. This is not all that surprising on reflection. In the physical environment, the properties of objects impose constraints on the types of actions to which they are amenable. In the LME, the properties of abstract objects will impose constraints on the epistemic actions to which they are amenable also. We will look at this in detail in section 5 below.

For now, note that with the example of DS above, the target proposition does not itself provide the antecedent of the relevant conditional types. Although we will see that this is true for many target proposition/inference rule pairings, it is not true in general. Consider *conjunction elimination* (CE). What CE tells us is that we may have a conjunct on its own, conditional on the conjunction of which the conjunct is a part being the target proposition.

That is, from $\phi \wedge \psi$ being the target propositions, which is to say the part of the LME to which we are attending, CE specifies that we are afforded ϕ and afforded ψ. The affordance of which we avail ourselves in practice will depend on the nature of the epistemic action that we perform. In affordance-type terms, CE is telling us that $\phi \wedge \psi$ is of multiple types. In particular, that $\phi \wedge \psi : (\phi \wedge \psi) \rightarrow \phi$ and $\phi \wedge \psi : (\phi \wedge \psi) \rightarrow \psi$. In contrast to DS, with CE the antecedent of the conditional type is *given to us* by the target proposition. Again, crucially, this does not mean that we are afforded the relevant conjunct directly. Rather, we must *act* on the target proposition in the way specified by the relevant conditional affordance type.

Before we say more about the epistemic actions at work, we need to identify a third and final affordance-type mechanism. With this done, we can complete an affordance-type taxonomy of inference rules in the following section. As an example of this third affordance-type mechanism, consider conditional introduction/*the rule of conditional proof* (CP). CP specifies an affordance type with respect to a target proposition, however, the nature of the affordance is *underspecified*. Unlike DS and CE above, the affordance type that is specified in practice will depend on the epistemic actions that one performs. With CP, the conditional affordance type is arrived at by a chain of epistemic actions that begin with the assumption that we have the antecedent as an affordance completely. This is an assumption that we are attending to a proper part of the LME via a factive mental state.

CP begins with a *hypothetical affordance* via a target proposition, ϕ say, which is then used to reach, via epistemic actions, another target proposition, ψ say. The result of this is the conditional affordance $\phi \rightarrow \psi$. Unlike the case with DS and CE, ψ may be anything at all. What matters with CP is that ψ was gotten to via a chain of epistemic actions that begin with ϕ. We might think of CP usefully as a method by which we may *discover affordance types*. Put another way - our epistemic deficit might be with regard to the topography of the LME itself, and not merely with regard to target proposition considered *in situ*. CP is a way for us to discover affordance types via discoveries about said topography.

Having made a case for the claim that inference rules are specifications of environmental affordance types, in the following section we will test this claim against a system of natural deduction.

4 Inference rules and affordance types

Consider a system of natural deduction, N_1 (Smith, 2012). Here we have a intro/elim rule pair for implication, conjunction, negation, and disjunction. We will start with the intro/elim pair for *implication*.

The introduction rule for implication in N_1 is simply the rule of conditional proof (CP) above. Since we have dealt with it already, we will move directly to *conditional elimination*. The elimination rule for the conditional in N_1 is simply *modus ponens* (MP). The affordance type specified by MP with regard to a target proposition ϕ, is $\phi : (\phi \rightarrow \psi) \rightarrow \psi$. With regard to $\phi \rightarrow \psi$, the affordance type specified by MP is *identified by the target proposition completely*. That is, $\phi \rightarrow \psi$ just *is* an affordance type.

The affordance type specified *conjunction introduction* (CIn)[3] with respect to ϕ is as follows - $\phi : \psi \rightarrow (\phi \wedge \psi)$. The important thing here is to note that a conjunction is a mere *aggregation* action, as opposed to the the *combinatorial* type actions underpinning MP say. We say more about the distinction between aggregation and combinatorial actions in the following section. The elimination rule for conjunction in N_1 has been dealt with in the section above, so we will move directly the intro/elim pair for *negation*.

The affordance type specified by *negation introduction* (NI) operates similarly to that of conditional elimination/conditional proof (CP) described in the preceding section above. Like CP, NI begins with an assumption that we have reached the relevant part of the LME, ϕ say, which is then used to reach, via epistemic actions, a pair of target propositions, one of which is the negation of the other. This pair will then return the negation of the hypothetical affordance. That is, $\phi : (\psi \wedge \neg\psi) \rightarrow \neg\phi$. *Negation elimination* (NE) is typed similarly, as $\neg\phi : (\psi \wedge \neg\psi) \rightarrow \phi$, again with the specification that $(\psi \wedge \neg\psi)$ is arrived at via a sequence of epistemic actions from the hypothetical affordance $\neg\phi$. More generally, we might think of pairs of inconsistent propositions as *universal affordances* such that they afford the opportunity to attend to any target proposition in the LME whatsoever.

The affordance type specified by *disjunction introduction* (DI) on the basis of the target proposition ϕ is one where the target proposition itself provides the antecedent of the relevant conditional type. That is, $\phi : \phi \rightarrow (\phi \vee \psi)$. The affordance type specified by *disjunction elimination* (DE) on the basis of the target proposition $\phi \vee \psi$ is complicated in comparison. Slightly differently to CP and NI, DE proceeds on the basis of *two distinct*

[3] We write "CIn" instead of the intuitive "CI" here in order to avoid confusion with the structural rule of Weak Commutation in Section 6 below.

hypothetical affordances, with one of each corresponding to one of each of the disjuncts of the target proposition $\phi \vee \psi$ itself. If from each of these hypothetical affordances ϕ and ψ, the same target proposition γ can be reached via a series of epistemic actions, then γ is reached definitively from the original target disjunction $\phi \vee \psi$. Hence DE types disjunction targets as $\phi \vee \psi : ((\phi \rightarrow \gamma) \wedge (\psi \rightarrow \gamma)) \rightarrow \gamma$.

Although the claim that inference rules are specifications of environmental affordance types is still hardly uncontroversial, I hope that enough has been demonstrated so far for it to be plausible at least. With this hope in mind, in the following section we will discuss epistemic actions on their own terms and in detail.

5 Epistemic actions

Epistemic actions have been doing some heavy lifting so far, so something substantial had best be said about them. As noted above, an epistemic action is any action performed in order to alleviate an epistemic deficit. This much is uncontroversial.

Although we do not pretend to anything like a complete taxonomic breakdown of such actions and deficits, we can say the following. If you suffer from an epistemic deficit such that the epistemic route to alleviating that deficit is *a posteriori*, then performing the relevant observation - a canonical example of an epistemic action if there is such a thing - might likely relieve the relevant deficit itself.[4]

We find ourselves in such deficits on account of our failing to be omniscient. If we were omniscient, then epistemic actions of the above sort would not alleviate our epistemic deficits about empirical matters of fact for the simple reason that we would not be in any such deficits in the first place.

If a route to alleviating an epistemic deficit is *a posteriori*, then we will be acting in the concrete physical domain by definition. Many routes to alleviating epistemic deficits are *a priori*, and the epistemic actions that take us along such routes will not be empirically directed actions such as observations or announcements. *a priori* routes are travelled by acts of reason,

[4]For a trite but useful example, if you do not know how much money is in your wallet, and you know that you suffer from this deficit and you want to alleviate it, then looking in your wallet should, barring exceptional accident, do the trick. Of course plenty of epistemic deficits whose route to relief is *a posteriori* might not lend themselves to observations for resolution in practice, if only because they are in the past. In such cases *testimony/announcement* actions will be playing a crucial role. I shall not discuss such cases here.

acts of the mind. These latter epistemic actions are acts of the mind on its own states, and it is via such inferential epistemic actions that we traverse the abstract environment of logico-mathematical structures.

"Inference" is very broad, but it captures the important fact that we can and do recognise that we suffer from epistemic deficits from which recovery requires us to think about things. Thinking is an act of the mind, but no less an action for this. We find ourselves in the relevant epistemic deficits here on account of our not being *logically omniscient*. If we *were* logically omniscient, then reasoning-based epistemic actions could never alleviate the relevant epistemic deficits for the simple reason that we would not be in any such deficits in the first place.

To reemphasise the running theme so far, inference *rules* specify the epistemic actions afforded to us by the parts of the LME to which we are attending. The part of the LME to which we are attending is the part that constitutes the target proposition of some propositional attitude of ours. In keeping with the terminology popular in theoretical computer science (if not mainstream philosophy), we will call *any* mental state of an agent underpinning such propositional attitudes "epistemic".

Although we do not pretend to anything like a complete taxonomic breakdown of epistemic actions of this latter psychological sort, we can make some headway. One such psychological epistemic action is something that we might, tentatively, call *aggregation*. Consider the case where you bear a propositional attitude, call it A, towards some proposition ϕ, written $A\phi$. In this case you are in a state of mind, you bear some particular mental state m, that is directed towards, or is about, ϕ itself, or takes ϕ as its object. In this case we say that ϕ is the part of the LME to which we are attending.

Suppose that you bear another instance of this same propositional attitude type towards some other proposition ψ, hence $A\psi$. In this case you bear some mental state m' that is directed towards ψ. In this case ψ is the part of the LME to which we are attending. It does not follow from these facts alone that $m = m'$. This is just to say that bearing a propositional attitude towards a proposition and bearing another instance of that same propositional attitude towards another proposition in no way entails that you bear a single instance of that propositional attitude to both of these propositions taken together. This is to say no more than that a careful account of propositional attitudes will not understand them to be closed under conjunction. From $A\phi$ and $A\psi$ it does not *follow* that $A(\phi \wedge \psi)$ for the simple reason that you might not have borne a mental state m'' that takes the conjunction of ϕ and ψ as its object. That one has attended to the part of the LME populated by ϕ, and that one has attended to the part of the LME populated by ψ, does not imply that one has attended to the part of the LME populated by $\phi \wedge \psi$. To get there requires labour on our part - an *aggregative* epistemic action.

In spite of $A(\phi \wedge \psi)$ not being a mere logical consequence of $A\phi$ and $A\psi$, $A(\psi \wedge \phi)$ is still an attitude that you might achieve on the basis of $A\psi$ and $A\phi$, *along with* some mental effort on your part. Again, the mental effort will comprise an epistemic action of the aforementioned psychological sort, what we are calling *aggregation*. Aggregation actions are those that we perform in order to bring together within the scope of a single instance of a propositional attitude those propositions that were previously within the scope of distinct instances of propositional attitudes. Although there is no restriction in principle that all attitudinal instances in such cases be of the same type - I might through aggregations come to know that I both believe that ϕ and desire that ψ say, in practice we will consider aggregation actions that operate on instances of epistemic attitudes of the same type only.

Aggregation is labour, and like any act requiring labour on our part, it is prone to error. Again, we are not logically omniscient, and neither are what we might call *maximally psychologically introspective*. Our mental states are not transparent to us, and neither do we possess infallible memories of facts in general, of which our previously transparent mental states are a proper subset. Hence:

Slogan 6 *We can get lost in the LME, and have accidents too.*

A different psychological epistemic action is that which we will call *combination*. The result of a successful aggregation action is a mental state that is no greater than the sum of the parts of mental states that the action aggregated. Combination actions, by contrast, are *generative* actions *on* the contents of mental states such that the results of such actions *are* greater than the sum of their parts. This is a fine distinction. Consider the following example. You ask a new logic student to consider or entertain (we take *entertaining* to be a non-committal propositional attitude) $\neg\psi$ and $\phi \rightarrow \psi$. In this case both $\neg\psi$ and $\phi \rightarrow \psi$ are aggregated within the scope of a single attitude of the student's. You then ask the student what, if anything, follows from this pair of propositions. The mere aggregation is insufficient for the student to answer correctly, that is for the student to bear the belief or knowledge attitude towards the claim the $\neg\phi$ (or any other logical consequence beyond aggregation for that matter).

Anyone who has taught introductory logic to a large cohort knows just how counterintuitive *modus tollens* is to many students on their first encounter.

In spite of having aggregated the premises and being in a mental state m that is directed towards the conjunction of the relevant par of propositions, this aggregation alone is insufficient for the student to be give a novel answer, to move to a mental state m' such that m' is directed towards $\neg\psi$. When confronted with such a pair of propositions, students answer often with "I don't know". In order for the student to move from m to m', they need to *combine* the propositions borne by m and m', and combine them in the right way.

Before we look more closely at combination actions themselves, especially at how we might say something philosophically robust with regard to what comprises combining the contents of mental states in "the right way", we will say something about the relationship between aggregation actions and combinations actions.

The first thing is that aggregation is a necessary condition on combination. This does not seem too controversial. If I am going to *combine* the propositions towards which a pair of my mental states are directed, I need to aggregate this pair before any such combination action can be performed. Aggregation actions are not sufficient for combination actions however, as the *modus tollens* example above demonstrates.

Aggregation actions can stand as necessary conditions on combination actions in a second manner that is distinct to that described above. Suppose that you bear some attitude A towards $(\phi \wedge \psi) \rightarrow \gamma$, and suppose also that you are looking for a way to perform *modus ponens* in order to discharge the consequent γ. In this case you will need to perform an aggregation action on ϕ and ψ in order to get $\phi \wedge \psi$ so that the conjunction may be used as a second premise in order to be able to perform the relevant consequent discharge. In other words, you might need to perform an aggregation action in order to form the very thing that will be one of the components in a combination action. In yet *other* words, *you might need to avail yourself of several affordances in order to get to where you want to be*.

The next thing that we note about our action pair is that there is a sense in which aggregation actions can never fail. They can fail to be realised, but when realised they are never illegitimate. This is not to say that they can never be *false*. Some aggregation actions will result in aggregations such as $\phi \wedge \neg\phi$, which are always false. The point is rather that the aggregation action itself is not illegitimate. Even in the case of explicit contradictions, we aggregate such things for the purposes of demonstrating explosion, or when we are explaining contradictions themselves. If the attitude under which such aggregation is taking place is one that comprises assent, such as belief, then the resulting aggregation will result in a mental state that is in error, but this error *depends* on the aggregation having taken place.

In contrast to aggregation actions, combination actions *can* be illegitimate. They can fail outrightly in the sense that a combination may be attempted that simply cannot achieve its goal. Suppose that I am trying to perform *modus ponens* on $\phi \rightarrow \psi$ and γ. Any attempt to do so will be an outright failure. An attempt to combine $\phi \rightarrow \psi$ and γ will not result in anything at all. Importantly, we should not be tempted into thinking that it will result in $(\phi \rightarrow \psi) \wedge \gamma$. The latter is the result of an aggregation action, not a combination action. The aggregation action must have been performed successfully in order for the (doomed, tragically) combination action to be attempted in the first place. There is simply no mental state m that is an accurately developed state on the basis of an illegitimate combination action. By analogy, they are akin to functions given an input that they do not accept, or programs being fed data of the wrong sort. There is simply no output at all, *because of a misidentification of affordance types with respect to inference rules*. Aggregation actions are un-typed instances of CI, whereas combination actions of the sort that we are discussing presently are typed instances of CE! With regard to the target proposition $\phi \rightarrow \psi$, what CI tells us is that $\phi \rightarrow \psi : \gamma \rightarrow ((\phi \rightarrow \psi) \wedge \gamma)$. By contrast, CE tells us that $\phi \rightarrow \psi : \phi \rightarrow \psi$.

There is a strongly normative flavour to this story, because in actual practice I might reason badly and possess false beliefs about the veracity of my combination actions and their resulting mental states. I might have false beliefs about just what actions are afforded to me by my local environment. Good. As stated at the beginning of the paragraph above, we are trying to say something philosophically robust about what it means for combination actions to be performed in the right way.

Whether a combination action is legitimate and successful or not will depend on both the logical form of the proposition towards which one's attitude is directed, as well as the inference rule that one is attempting to apply. That is, *their success will depend on one's availing one's self of the correct affordance given the target proposition that is the object of one's propositional attitude*.

6 Modelling combination actions/affordance actualisation

We want a formal model theoretic structure that allows a natural interpretation in terms of attitudinal states and psychological combination actions of the sort introduced in section 5 above. To this end we introduce a frame **F** : $\langle S, \bullet \sqsubseteq \rangle$, where S is a set of information states $x, y, z \ldots$, $\bullet$ is a binary composition operator on members of S, and $\sqsubseteq$ is a partial order on S. A model **M** : $\langle \mathbf{F}, \Vdash \rangle$, where $\Vdash$ is a relation between members of S and propositions $A, B, C \ldots$.

We need to give our model a robust attitudinal interpretation. To this end, we take the domain of S to be a set of attitudinal states of an agent α. Although we think that the proposal below is general enough to apply to attitudinal states of any type, in practice we will limit our discussion to attitudes involving assent, such as doxastic or epistemic attitudes.

In this case we may read $x \Vdash A$ as *α knows/believes that A* - or equivalently - *α is attending to the part of the LME populated by A*. Importantly, we place a restriction on **M** such that we understand $x \Vdash A$ to mean that A is the *only* part of the LME constituting α's state x.

$x \bullet y$ is understood as the combination of α's attitudinal states x and y by α themselves, as an explicit psychological action.

$x \sqsubseteq y$ indicates informational-relevance in general, and explicit attitudinal relevance in particular. So if we are interpreting S as a set of epistemic states, $x \sqsubseteq y$ will be read as *x is epistemically relevant to y*, and $x \bullet y \sqsubseteq z$ as *the act of combining x and y is epistemically relevant to z*. We may also read $x \sqsubseteq y$ as *the part of the LME that is the target of α's epistemic state x is contained in the part of the logico-mathematical environment that is the target of α's epistemic state y.*

We can say the following about attitudinal relevance/LME containment. Firstly, any attitudinal state will be relevant to itself, hence $\forall x x \sqsubseteq x$. Similarly, any part of the LME will be a part of itself. Secondly, the relevance of an epistemic action $x \sqsubseteq y$ to some epistemic state y will depend on the logical form of the propositions towards which the attitudes x, y, and z are directed. For example, if $x \Vdash A \rightarrow B$ and $y \Vdash A$ and $z \Vdash B$, then $x \bullet y \sqsubseteq z$, but $y \not\sqsubseteq z$. The latter is the case because A is not informationally relevant to B on its own, not without further context which is given in this case by x. Similarly, we may say that A is not a proper part of the LME that is populated by B, at least not without further environmental context.

What we need is some way of specifying constraints on our action operation $\bullet$ such that these constraints will preserve the epistemic relevance of the corresponding psychological action. Put another way, we want a way of specifying properties of $\bullet$ so that these properties guarantee that the result of performing the successful psychological epistemic action is preserved. Put yet a third way, we want some way of guaranteeing progress across the logic-mathematical environment.

We can specify such constraints (and *ipso facto* permissions) with the familiar structural rules of substructural logic. Via (Restall, 2000), 250, we list the frame conditions for the most common structural rules (B) Associativity, (B^c) Converse Associativity, (B') Twisted Associativity, (C) Commutation, (CI) Weak Commutation, (W) Contraction, (WI) Weak Contraction, (M) Mingle, (K) Weakening, and (K') Commuted Weakening below (reading $\Rightarrow$, $\wedge$, and $\vee$ as "if then", "and", and "or" in the metalanguage respectively):

$$\exists u((x \bullet y \sqsubseteq u) \wedge (u \bullet z \sqsubseteq w)) \Rightarrow \exists u((y \bullet z \sqsubseteq u) \wedge (x \bullet u \sqsubseteq w)) \quad \text{(B)}$$

$$\exists u((y \bullet z) \sqsubseteq u) \wedge (x \bullet u \sqsubseteq w)) \Rightarrow \exists u((x \bullet y \sqsubseteq u) \wedge (u \bullet z \sqsubseteq w)) \quad (\text{B}^c)$$

$$\exists u((y \bullet x \sqsubseteq u) \wedge (u \bullet z \sqsubseteq w)) \Rightarrow \exists u((y \bullet z \sqsubseteq u) \wedge (x \bullet u \sqsubseteq w)) \quad (\text{B}')$$

$$\exists u((x \bullet z \sqsubseteq u) \wedge (u \bullet y \sqsubseteq w)) \Rightarrow \exists u((x \bullet y \sqsubseteq u) \wedge (u \bullet z \sqsubseteq w)) \quad \text{(C)}$$

$$(x \bullet y \sqsubseteq z) \Rightarrow (y \bullet x \sqsubseteq z) \quad \text{(CI)}$$

$$(x \bullet y \sqsubseteq z) \Rightarrow \exists w((x \bullet y \sqsubseteq w) \wedge (w \bullet y \sqsubseteq z)) \quad \text{(W)}$$

$$x \bullet x \sqsubseteq x \quad \text{(WI)}$$

$$(x \bullet x \sqsubseteq y) \Rightarrow (x \sqsubseteq z \vee y \sqsubseteq z) \quad \text{(M)}$$

$$(x \bullet y \sqsubseteq z) \Rightarrow x \sqsubseteq z \quad \text{(K)}$$

$$(y \bullet x \sqsubseteq z) \Rightarrow x \sqsubseteq z \quad (\text{K}')$$

We give the evaluation conditions for two instances of our combination action operator $\bullet$. The first where a conditional is being combined with a potential input, the second where conditionals themselves are being combined. This is just to say that the first is where *affordances are being actualised*, and the second is where *affordances are being composed*. Because we are talking about the properties of dynamic (epistemic) actions here, and not merely the results of the same, we must treat affordance-type operations intensionally:

$$x \Vdash A \to B \text{ iff } \forall x \forall y : x \bullet y \sqsubseteq z, \text{ if } y \Vdash A \text{ then } z \Vdash B \quad (1)$$

$$x \Vdash A \to B \text{ iff } \forall x \forall y : x \bullet y \sqsubseteq z, \text{ if } y \Vdash B \to C \text{ then } z \Vdash A \to C \quad (2)$$

CLAIM 1 Combination actions of type (1), *affordance actualising actions*, are guaranteed to have their success preserved by (CI), but destroyed by all other structural rules.

CLAIM 2 Combination actions of type (2), *affordance composing actions*, are guaranteed to have their success preserved by (B), (B^c), but destroyed by all other structural rules.

We start by justifying CLAIM 1. Consider (CI). Given that the action here is combining a conditional with its antecedent, the success of this action is order-invariant, so (CI) holds. The logical form of the propositions being combined in *affordance actualisation* type actions forces the discharge of the consequent. There is only one way that things can go, so to speak.

We should reemphasise the restriction on **M** outlined above, such that we understand $x \Vdash A$ to mean that A is the *only* part of the LME constituting α's state x. In the more abstract terms of our model *qua* model, we take *satisfaction* to be a primitive notion, there to be no points/information states other than those mentioned, and that they satisfy or support exactly and only the formulas that they are stated to satisfy.[5]

To see why it is that combination actions of type (1) have their success destroyed by the other structural rules, we start with (B). Suppose that $x \Vdash A \rightarrow B, y \Vdash A, u \Vdash B, w \Vdash C$, and $z \Vdash B \rightarrow C$. In this case the antecedent of (B) is satisfied whist the consequent is false. The consequent is false on account of its left hand conjunct being false. The is no u such that it is the result of $y \bullet z$. Why is this? Recall the restriction on *M* above - that we understand $x \Vdash A$ to mean that A is the *only* part of the LME constituting α's state x. our models here are not general substructural models, but rather those in which certain states are *identified* with certain formulas. Syntactically speaking, attempting to discharge the consequent from $B \rightarrow C$ by combining it with A is doomed. Note that the argument against (B) depends on the argument for (CI) above. If (CI) were not acceptable then the antecedent of (B) would not be satisfied, on account of $u \bullet z \not\sqsubseteq w$ in this case.[6]

Now consider (B^c), and suppose that $y \Vdash A, z \Vdash A \rightarrow B, u \Vdash b, w \Vdash C$, and $x \Vdash B \rightarrow C$. In this case the antecedent of (B^C) will be satisfied whilst the consequent is false, again on account of the consequent's left hand conjunct being false for reasons similar to those concerning the left hand conjunct of the consequent of (B).

Now consider (B'), and suppose that $y \Vdash A, x \Vdash A \rightarrow B, u \Vdash B, z \Vdash B \rightarrow C$, and $w \Vdash c$. Here the antecedent of (B') will be satisfied whilst its

[5] I am indebted both to Igor Sedlar and to an anonymous referee for making me be clearer on this point than I would have been had I been left to my own devices.

[6] I am indebted to the same anonymous referee for making me be clearer here.

consequent will be false on account of its consequent's left hand conjunct being false for reasons analogous to those given above.

Now consider (C), and suppose that $x \Vdash A, z \Vdash A \rightarrow B, u \Vdash B, y \Vdash B \rightarrow C,$ and $w \Vdash C$. In this case the antecedent of (C) will be satisfied, but the left hand conjunct of its consequent will be false, hence its consequent will be false.

Now consider (W), and suppose that $x \Vdash A \rightarrow B, y \Vdash A,$ and $z \Vdash B$. In this case there is no way to satisfy both of the consequent's conjuncts. We can satisfy the left hand conjunct with $w \Vdash B$, in which case the right hand conjunct will be false. Alternatively, we could satisfy the right hand conjunct with $w \Vdash A \rightarrow B$, but now the left hand conjunct will be false.

Now consider (WI). (WI) is not a conditional. It states that any combination of an information state with itself develops into that same information state. In our epistemic attitudinal gloss, the combination action of an epistemic state with itself is epistemically relevant to that same state. This is false no matter what we choose for x. Speaking syntactically, combining (as opposed to aggregating) any proposition with itself is a doomed attempt at epistemic advancement.

Now consider (M), and suppose that $x \Vdash A \rightarrow B, y \Vdash$, and $z \Vdash B$. In this case the antecedent of (M) will be satisfied but the consequent is false. We have it that $x \bullet y \sqsubseteq z$, however it is neither the case that $x \sqsubseteq z$, nor $y \sqsubseteq z$. With regard to the former, $A \rightarrow B$ is not non-contextually epistemically relevant to B (we need A as further context). With regard to the later, A is not non-contextually epistemically relevant to B, as here too we need further context.

Now consider (K), and suppose that $x \Vdash A \rightarrow B, y \Vdash A,$ and $z \Vdash B$. In this case the antecedent of (K) will be satisfied but the consequent will be false. The consequent states that $x \sqsubseteq z$, but this is not the case as speaking syntactically $A \rightarrow B$ is not non-contextually epistemically relevant to B. Any attempt to discharge B from $A \rightarrow B$ without A is doomed. The reasoning with regard to (K′) is identical to that surrounding (K).

We now justify CLAIM 2. Consider (B). There is no way of satisfying the antecedent that will make the consequent false. As an illustrative exercise, suppose that $x \Vdash A \rightarrow B, y \Vdash B \rightarrow C, u \Vdash A \rightarrow C, z \Vdash C \rightarrow D,$ and $w \Vdash A \rightarrow D$.

The reasoning with regard to (B^C) is identical. This is not surprising, since conditionals can be thought of as functions, and function composition is associative, and association is meant often in its biconditional form, which is just the conjunction of (B) and (B^C).

Now consider (B′), and suppose that $y \Vdash A \rightarrow B, x \Vdash B \rightarrow C, u \Vdash A \rightarrow C, z \Vdash C \rightarrow D$, and $A \rightarrow D$. In this case the antecedent will be satisfied but the consequent will be false. The consequent is false on account of both of its conjuncts being false. Speaking syntactically, attempting to compose either $A \rightarrow B$ with $C \rightarrow D$, or $B \rightarrow C$ with $A \rightarrow C$ is doomed.

Now consider (C), and suppose that $x \Vdash A \rightarrow B, z \Vdash B \rightarrow C, u \Vdash A \rightarrow C, y \Vdash C \rightarrow D,$ and $w \Vdash A \rightarrow D$. In this case the antecedent will be satisfied by the left hand conjunct of the consequent false for reasons similar to those surrounding (B′) above.

Now consider (CI), and suppose that $x \Vdash A \rightarrow B, y \Vdash B \rightarrow C,$ and $z \Vdash A \rightarrow C$. Here the antecedent will be true but the consequent will be false. Speaking syntactically, trying to feed the consequent of $B \rightarrow C$ to the antecedent of $A \rightarrow B$ is the wrong order for cutting out the middle or joining proposition.

Now consider (W), and suppose that $x \Vdash A \rightarrow B, y \Vdash B \rightarrow A,$ and $z \Vdash A \rightarrow A$. In this case the consequent will be false as there is no w that will satisfy both conjuncts. Either $w \Vdash A \rightarrow A$ in which case the right hand conjunct is false, or $w \Vdash A \rightarrow B$ in which case the left hand conjunct is false.

Now consider (WI). The reasoning here is identical to that surrounding (WI) for CASE 1 above.

Now consider (M), and suppose that $x \Vdash A \rightarrow B$, $y \Vdash B \rightarrow C$, and $A \rightarrow C$. Here the antecedent of (M) will be satisfied whilst its consequent will be false. Although we do have it that $x \bullet y \sqsubseteq z$, we have it neither that $x \sqsubseteq z$, nor that $y \sqsubseteq z$. With regard to the former, it is not the case that $A \rightarrow B$ is non-contextually epistemically relevant to $A \rightarrow C$. With regard to the latter, it is not the case that $B \rightarrow C$ is non-contextually epistemically relevant to $A \rightarrow C$.

Now consider (K), and suppose that $x \Vdash A \rightarrow B, y \Vdash B \rightarrow C,$ and $z \Vdash A \rightarrow C$. In this case the antecedent will be satisfied but the consequent false. x is not informationally relevant to z all on its own, that is non-contextually. Speaking syntactically, trying to get $A \rightarrow C$ from $A \rightarrow B$ without any other informational artefacts or actions is doomed. The reasoning with regard to (K′) is identical to that surrounding (K).

This completes our proposal for a logic of affordances. We have seen that combination actions of types corresponding to *affordance actualising actions*, have their success preserved by (CI), but destroyed all other structural rules. We have seen also that combination actions of types corresponding to *affordance composing actions*, have their success preserved by (B), (B^c), but destroyed by all other structural rules.

7 Conclusion

We have covered a lot of ground above, and I do not pretend to anything like confidence that I will have convinced all readers. My hope is that the above is worked out sufficiently for it to strike most as plausible, and worthy of further pursuit.

I have made a case for the abstract LME affording opportunities for action in ways not dissimilar entirely from the the ways in which the concrete physical environment affords the same. I have motivated, or tried to motivate at least, an understanding of inference rules as specifications of affordance types, such that they afford us opportunities for successful action across abstract environments. I have proposed that such actions be understood properly as epistemic actions of a psychological sort, and I have put forward and argued for a weak substructural logic as a plausible model for said affordance types and their related epistemic actions (or more strictly, a unique weak substructural logic for discrete affordance-actions).

The frame conditions given in (1) and (2) in section 6 above correspond the serial and parallel composition of information channels in channel theory. Given the by now, and increasingly, well known correspondence between channel theory and the ternary frame semantics of relevance and substructural logics (Mares, 1996), Restall (1996), this is not itself a surprise. Indeed the modelling of channel-theoretic phenomena in frame semantics terms has led to a recent revival of interest (Tedder, 2017). What might be a surprise is that a channel-theoretic interpretation of the subject matter at hand is both straightforward and natural. The insight motivating channel-theory in the first place is that one part of our environment may carry information to, or about, another part of it. Recall also the initial promise of situation theory to contribute to an analysis of hyperintensional phenomena and mathematical knowledge (Barwise and Perry, 1983). A robust platonism of the sort proposed above allows for a sensible mapping of this insight from the concrete environment over to the abstract environment of logico-mathematical objects. In a final slogan:

Slogan 7 *Affordances are information channels.*

References

Aucher, G. (2014). Dynamic Epistemic Logic as a Substructural Logic, in A. Baltag and S. Smets (eds.): *Johan van Benthem on Logic and Information Dynamics (Outstanding Contributions to Logic 5)*, Cham: Springer.2014, pp. 855-880.

Barwise, J and Perry, J. (and John Perry,)1983). *Situations and Attitudes*, MIT Press, Cambridge, MA.

Bermudez, J. L. (1998). *The Paradox of Self-Consciousness*,MIT Press, Cambridge, MA.

Gibson, J. J. (1966). *The Senses Considered as Perceptual Systems*, Houghton Mifflin, Boston.

Restal, G. (2000). *An Introduction to Substructural Logics*, Routledge.

Restall, G. (1996). Information flow and relevant logics, in J. Seligman and D. Westerstahl (eds.), *Logic, Language and Computation*, CSLI Publications, Stanford. pp. 463-477, 1996.

Rowlands, M. (1997). ‘Teleological Semantics’, *Mind*, 106(422): 279 – 303.

Sedlar, I. (2015). Substructural Epistemic Logics, *Journal of Applied Non-classical Logics*, 25(3): 256-285.

Smith, N. J.J. (2012). *Logic: The Laws of Truth*, Princeton University Press.

Tedder, A. (2017). Channel Composition and Ternary Relation Semantics, in *IFCoLog Journal of Logics and Their Applications*, v.4, n. 3, pp. 731-753.

Mares, E. (1996): Relevant Logic and the Theory of Information, *Synthese*, 109, no. 3, 345–360.

Williamson, T. (2002): Necessary Existents, In A. O’Hear (ed.), *Royal Institute of Philosophy Supplement*, Cambridge University Press, pp. 269-87 (2002).

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